An Introduction to Mathematical Finance — Exercises 1
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Time value of money

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1. $r \simeq 0.0133 = 1.33\%$ and $R(0, 61/365) \simeq 0.0022 = 0.22\%$.
2. $t \simeq 0.4167$ years or $t \simeq 152.096$ days. $R(0,t) \simeq 0.0375 = 3.75\%$.
3. $P \simeq 98,044.5$ NOK.
4. $t \simeq 11.5534$ years or roughly 11 years and 202 days.
5. a) $V(2) = 121,000$ NOK and b) $V(2) = 121,551$ NOK.
6. a) $V(0) \simeq 674.025$ NOK and b) $V(0) = 760.449$ NOK.
7. $R(0,1) = \left(1+\frac{r}{m}\right)^m > 1+r$, when $r > 0$ and $m \geq 2, m \in \mathbb{N}$. It can be proved by expanding $R(0,1)$ using the binomial formula.
8. In general, let assume N years, interest rate r and amount borrowed P. Then the formula for each instalment is

$$C = \frac{rP}{1-(1+r)^{-N}}.$$

The outstanding balance remaining after $n-1$ instalments is

$$P\frac{(1+r)^N - (1+r)^{n-1}}{(1+r)^N - 1}.$$

The interest included in the nth instalment is

$$P\frac{(1+r)^N - (1+r)^{n-1}}{(1+r)^N - 1}r.$$

The capital repaid as part of the nth instalment is

$$P\frac{r(1+r)^{n-1}}{(1+r)^N - 1}.$$

9. $V(0) \simeq 449,409$ NOK.

10. $V(40) \simeq 1.4496 \times 10^6$ NOK.

11. The present value of n such payments is given by $C\frac{1-\left(\frac{1+g}{1+r}\right)^n}{r-g}$ and the present value of an infinite stream of such payments is given by $C\frac{1}{r-g}$, provided $g < r$.

12. Setting 1626 as $t = 0$, a) $V(392) \simeq 7.80518 \times 10^9$ and b) $V(392) \simeq 4.85754 \times 10^9$.

13. $r \simeq 0.102587 \simeq 10.26\%$.

14. $r = 0.18 = 1.8\%$.

15. $r \simeq 0.1194 = 11.94\%$.

16. $V(0) = C\frac{1-(1+r_e)^{-n}}{(1+r_e)^{1/12}-1}$.

17. The implied annual compounding rate is $r_1 \simeq 0.118605 \simeq 11.86\%$, the implied semi-annual compounding rate is $r_2 \simeq 0.115283 \simeq 11.53\%$ and the implied continuous compounding implied rate is $r_c \simeq 0.112083 \simeq 11.20\%$.

18. $R(0, 75/365) \simeq 0.0242 \simeq 2.42\%$.

19. $r \simeq 0.1353 = 13.53\%$.

20. $t = 0.5851$ years or 213.6 days.