An Introduction to Mathematical Finance — Exercises 2

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Basic Financial Derivatives

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1. Yes there is an arbitrage opportunity. It is implemented as follows:
   At time 0:
   - Enter a long forward contract with forward price NOK180.
   - Short-sell 1 stock and get $S(0) = 170$.
   - Invest 70% of $S(0)$ risk free at 0.08 interest rate.
   - Put 30% of $S(0)$ in the security deposit attracting interest at 0.04.

   At time T use the forward contract to buy the asset for NOK180 and return it to the owner. Close the security deposit and the risk free investment. This strategy gives you a positive riskless profit.
   The highest rate d for which there is not arbitrage opportunities is $d = 0.00174$.

2. The percentage drop is 11.34%.

3. This exercise has been solved in class.
   (a) The strategies used to prove that if the formula does not hold there is arbtitrage are similar to the case without dividends. You ony need to invest/borrow the dividend D from time t_D until T, depending if in the strategy that you are implementing you hold/have sold the stock.
   (b) The idea is the same as in the previous section. However this time you buy/sell-short $e^{-r_D(T-t)}$ fraction of the stock and invest/borrow the dividends to increase progressively your position in the stock to 1 share at time T.

4. If the inequality holds then the following strategy leads to an arbitrage:
   At time t:

- Enter a short forward position with forward price $F(0,T)$, by receiving $V(t)$.
- Invest $V(t)$ at the risk-free rate.
- Enter a long forward position with forward price $F(t,T)$ at zero cost.

At time T close the forward positions and the position in the money market. This strategy gives you a positive riskless profit.

5.
   (a) With $S(3/4) = 490$ we get $V(3/4) = 39.5867$.
   (b) With $S(3/4) = 450$ we get $V(3/4) = -0.4137$.

6. $S(1/365) = S(0)\, e^{r/365}$.

7. $S(T) = 283.63$.

8. The expected gain is -5.37.

9. $P^E = 19.8$.

10. The arbitrage opportunity follows from a violation of the put-call parity. It is implemented as follows:
At time 0:

   - Sell a call for $C^E = 50.9$.
   - Buy a put for $P^E = 77.8$.
   - Buy a share for $S(0) = 203.7$.
   - Borrow $S(0) - C^E + P^E = 230.6$.

   At time T=1/2 you can sell the share for 240 either using the put option or the call option. Repay the loan which amounts to $230.6 \times e^{0.0748 \times \frac{1}{2}} = 239.39$. This strategy gives you a positive riskless profit of 0.61.

11.
   (a) If $C^E < 0$ you "buy" the call receiving a positive amount that you invest risk-free. At time T you obtain a positive riskless profit form the money market investment and a non-negative amount form the option payoff. If $C^E > C^A$ you sell the European call and buy the American call at time zero. This gives you a positive initial amount that you invest risk-free, generating a positive riskless profit at time T. Note that you have bought the American option and you can decide to exercise the option at the same time as the European option. Hence, you get a zero final balance from the position in the options.
   (b) It is completely analogous to (a).

12. It is a matter of checking the formula for $X \leq K_1$,$K_1 < X \leq K$, $K < X \leq K_2$ and $X > K_2$. You can also prove it by using basic properties of convex functions and the fact that convexity is preserved by composing with a non-decreasing function.

13.

    (a) The same inequalities hold for an European call. Then, the result follows from the fact that $C^E = C^A$, proved in class.

    (b) The value of an American option must always be greater than its intinsic value, which is $K - S(0)$ for a put. As $P^A \geq 0$, we just have proved $P^A \geq (K - S(0))^+$. To prove that $P^A < K$, note that the put-call parity estimates for American options yield that

    $$P^A \leq C^A - S(0) + K < K,$$

    where in the last inequality we have used that $C^A < S(0)$,which we have proved in (a).

14. The strangle is created by buying both calls and it has an initial cost of 5. Assuming the interest rate $r = 0$, the profit table is given by

$S(T)$	Profit
$S(T) < 45$	$40 - S(T)$
$45 \leq S(T) < 50$	-5
$S(T) \geq 50$	$S(T) - 55$

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15. The resulting strategy is a butterfly spread. Let $K_1 < K_2$ be the two stikes prices of the strangle with initial cost of $C^E(K_2) + P^E(K_1)$ and $K = \frac{K_1+K_2}{2}$ be the strike of the (short) straddle that has initial profit of $C^E(K) + P^E(K)$. Assuming the interest rate $r = 0$, the profit table for the butterfly spread is given by

$S(T)$	Profit
$S(T) < K_1$	$\frac{K_1-K_2}{2} - \left(C^E(K_2) - C^E(K) + P^E(K_1) - P^E(K)\right)$
$K_1 \leq S(T) < K$	$S(T) - K - \left(C^E(K_2) - C^E(K) + P^E(K_1) - P^E(K)\right)$
$K \leq S(T) < K_2$	$K - S(T) - \left(C^E(K_2) - C^E(K) + P^E(K_1) - P^E(K)\right)$
$S(T) \geq K_2$	$\frac{K_1-K_2}{2} - \left(C^E(K_2) - C^E(K) + P^E(K_1) - P^E(K)\right)$