30$^{\text{th}}$ September, 2021

STK-MAT3700

Mandatory assignment 1 of 1

Submission deadline

Thursday 14$^{\text{th}}$ OCTOBER 2021, 14:30 in Canvas (canvas.uio.no).

Instructions

You can choose between scanning handwritten notes or typing the solution directly on a computer (for instance with LaTeX). The assignment must be submitted as a single PDF file. Scanned pages must be clearly legible. The submission must contain your name, course and assignment number.

It is expected that you give a clear presentation with all necessary explanations. Remember to include all relevant plots and figures. Students who fail the assignment, but have made a genuine effort at solving the exercises, are given a second attempt at revising their answers. All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, we may request that you give an oral account.

In exercises where you are asked to write a computer program, you need to hand in the code along with the rest of the assignment. It is important that the submitted program contains a trial run, so that it is easy to see the result of the code.

Application for postponed delivery

If you need to apply for a postponement of the submission deadline due to illness or other reasons, you have to contact the Student Administration at the Department of Mathematics (e-mail: studieinfo@math.uio.no) well before the deadline.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination.

Complete guidelines about delivery of mandatory assignments:

uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

GOOD LUCK!

To pass the assignment you need a score of at least 50p. All questions have equal weight.

Problem 1. In this problem we review some of the concepts in Lectures 2 and 3.

1. (10p) What is better, monthly compounding at 7.4% or semi-annually compounding at 7.5%?
2. (10p) How much can you borrow if the annual interest rate is 5%, you can afford to pay NOK10000 each month and you want to clear the loan in 25 years?
3. (10p) Let C^E, P^E, C^A, and P^A denote prices of a European call option, a European put option, an American call option and an American put option, respectively. All of them with expiry time T and the same strike price K. Let $r \geq 0$ be the continuously compounded interest rate. Show that:
   a) If
   $$C^E - P^E - S(0) + Ke^{-rT} < 0,$$
   then you can make a sure risk-less profit.
   b) If
   $$C^A - P^A - S(0) + Ke^{-rT} > 0,$$
   then you can make a sure risk-less profit.
4. (10p) A call option with strike price of NOK60 costs NOK6. A put option with the same strike and expiration date costs NOK4. Construct a table that shows the profit from a straddle. For what range of stock prices would the straddle lead to a loss? You may assume that the interest rate is zero.

Problem 2. Consider a single-period market consisting of a probability space $\Omega = \{\omega_1, \omega_2, \omega_3\}$, a probability measure $P(\omega) > 0, \omega \in \Omega$, a bank account with $B(0) = 1$, and $B(1) = 1$, and two risky assets, denoted by $S_1 = \{S_1(t)\}_{t=0,1}$ and $S_2 = \{S_2(t)\}_{t=0,1}$

$$S_1(0) = 7, \qquad S_1(1,\omega) = \begin{cases} 9 & \text{if} \quad \omega = \omega_1 \\ 7 & \text{if} \quad \omega = \omega_2 \\ 4 & \text{if} \quad \omega = \omega_3 \end{cases},$$

$$S_2(0) = 3, \qquad S_2(1,\omega) = \begin{cases} 3 & \text{if} \quad \omega = \omega_1 \\ 6 & \text{if} \quad \omega = \omega_2 \\ 3 & \text{if} \quad \omega = \omega_3 \end{cases}.$$

In this market :

1. (10p) Define dominant trading strategy and arbitrage opportunity. How are these concepts related?
2. (10p) Define linear pricing measure and risk neutral pricing measure. How are these concepts related?
3. (10p) What is the law of one price? How is it related with the previous concepts of pricing measures?
4. (10p) Does this market contain dominant trading strategies? Does it contain arbitrage opportunities? Check if the following strategies are dominant and/or arbitrage opportunities

   a) $H = (-6, 0, 2)^T$.
   b) $H = (-10, 1, 1)^T$.

Note: Adapt the general definitions to this particular market with two risky assets and three possible states of the economy.

Problem 3. Consider a single-period market consisting of a probability space $\Omega = \{\omega_1, \omega_2, \omega_3\}$, a probability measure $P(\omega) > 0, \omega \in \Omega$, a bank account with $B(0) = 1$, and $B(1) = 1 + r$, where $r \geq 0$ is a given interest rate, and one risky asset, denoted by $S_1 = \{S_1(t)\}_{t=0,1}$,

$$S_1(0) = 3, \qquad S_1(1, \omega) = \begin{cases} 4 & \text{if} \quad \omega = \omega_1 \\ 3 & \text{if} \quad \omega = \omega_2 \\ 2 & \text{if} \quad \omega = \omega_3 \end{cases} .$$

1. (10p) Determine the risk-neutral probability measures. Is the market free of arbitrage? Discuss the result in terms of the possible values of r.
2. (10p) What is the definition of a complete market? Is the market complete? Determine (characterize) the attainable claims. Discuss the result in terms of the possible values of r.
3. (10p) Set $r = 1/6$. Consider the contingent claim $X = (4, 7/2, 4)^T$. Determine the arbitrage-free prices of X.
4. (10p) Set $r = 1/6$. Assume that in the market is introduced a new risky asset S_2 with $S_2(0) = \frac{6}{7}$. Give conditions on $S_2(1) = (S_2(1, \omega_1), S_2(1, \omega_2), S_2(1, \omega_3))^T$ such that the extended market is complete. Check if $S_2(1) = (3/2, 1/3, 1/4)^T$ completes the market and, if this is the case, give the unique risk neutral measure.