1. Introduction

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Outline

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Outline of the Course

Some Basic Notions

Goal

- Introduce the most important concepts, principles and problems in modern mathematical finance.
- Theory of arbitrage for pricing and hedging derivatives in the context of discrete time models.
- Portfolio optimization and its connection to risk neutral pricing.
- Obtain the famous Black-Scholes option pricing formula (continuous time) as limit of discrete time valuation formulae.

Some basic notions

- ***Asset***: Anything of value. Assets can be risky or non-risky (riskless). Here, risk is understood as uncertainty that may cause losses (of wealth).
- ***Financial market***: A “place” where buyers and sellers exchange financial assets/ instruments/products/contracts. Security is also a synonym of financial asset.
- ***Portfolio***: A collection of financial assets.
- ***Traders***: Buyers and sellers in a financial market.

Some basic notions

Financial assets can be divided into two categories:

1. *Underlying/Primary assets*:
   1.1 **Shares**: Units of ownership interest in a corporation that provide for equal distribution in any profits in the form of dividends.
   1.2 **Bonds**: A debt financial asset, under which the issuer owes the holder a debt and is obliged to pay them interest (the coupon) and/or to repay the principal at a later date, termed the maturity date.
   1.3 **Commodities**: Oil, gas, precious metals, metals cocoa, coffee, sugar, pork...
   1.4 **Foreign currencies**: US Dollar, Euro, Yen,...
2. *Derivative securities*: Financial contracts that promise some payment (contingent claims) or delivery (forward commitments) in the future depending on the price of another financial asset (usually underlying assets).

Some basic notions

- The prices for underlying assets (and for very liquid/highly-traded derivatives) are set according to the law of supply and demand.
- The supply and demand are driven by different economic factors/considerations.
- Traders can use different economic theories to estimate the value of an asset, compare the estimated value with the asset price and decide if it is convenient to buy or sell the asset.

But for derivatives, the economic considerations to determine their value are less clear. For instance:

- Why there should be derivative securities at all?
- What is their economic motivation?

Inspirational Example (Hedging)

Hedging

- Suppose that you are a risk manager of a pension fund invested in the financial market.
- You know that in T years the fund needs to pay out K million NOK in retirement money to the pensioners.
- Assume that your portfolio consists in risky assets like shares.
- In T years, the value of your portfolio, denoted by $V(T)$, could be bigger than K million NOK, but it also could be smaller.

Problem

You face the risk that in T years your investment is worth less than the amount promised to the pensioners.

Hedging

Solution

- Enter into a financial contract with a counterparty that guarantees a minimal value of K million NOK for your portfolio in T years.
- Such contract gives you the right to sell your portfolio for K million NOK in T years, but if the portfolio is worth more you are no obliged to do so.
- Your counterparty, however, is committed to buy your portfolio for K million NOK if its market value in T years is worth less than K million NOK.
- This financial derivative is called a European put option.

Hedging

- The put option contract has the following payoff at time T,

$$\begin{aligned} P(T, V(T), K) &= (K - V(T))^{+} := \max(0, K - V(T)) \\ &= \begin{cases} K - V(T) & \text{if} \quad K \geq V(T) \\ 0 & \text{if} \quad K < V(T) \end{cases}, \end{aligned}$$

  the agreed price K is known as the strike price, while T is the exercise/expiry time of the option.

- At time T, you have

$$V(T) + P(T, V(T), K) = \begin{cases} K & \text{if} \quad K \geq V(T) \\ V(T) & \text{if} \quad K < V(T) \end{cases}.$$

Hedging

There are some issues:

- Asymmetry of the contract: If you enter into this contract you have a right while your counterparty has an obligation.
- At time T you will have a nonnegative payoff while your counterparty will have a nonpositive payoff.

Hence, your counterparty will not enter such a contract without receiving a ***premium*** from you.

Hedging

This raises two questions:

- What should this premium be in order for both of you accepting this deal as fair? (Pricing)
- How your counterparty will manage to, at time T, fulfill the agreement? (Hedging)

In principle this two questions do not seem to be related. However, we will see that they are intimately connected through the arbitrage pricing theory or the application of the principle of no arbitrage to pricing.

A Less Edifying Example (Speculation)

Speculation

- Suppose that the share price of a company is NOK1000 today.
- You are convinced that in one year the price will be NOK1200, but it well also may be NOK800.
- You invest NOK1000 today and:
  - If you are right in one year you will gain 20% of your initial investment.
  - If you are wrong you will lose 20% of your initial investment.
- There is an alternative strategy.
- Today, in the market, you can buy a European call option that guarantee you the possibility to buy in one year a share of the company for NOK1050, whatever the share price may be in one year.

Speculation

- The payoff of this derivative is

$$C(1, S(1), 1050) = (S(1) - 1050)^{+} = \max(0, S(1) - 1050)$$
$$= \begin{cases} 1200 - 1050 = 150 & \text{if} \quad S(1) = 1200 \\ 0 & \text{if} \quad S(1) = 800 \end{cases}.$$

- If you are right, in one year, this contract will give you NOK150.
- But if you are wrong it will give you nothing.
- To enter into this contract costs NOK50, so you decide to buy 20 contracts, $20 \times 50 = 1000$NOK.

Speculation

- Hence, if you are right you will make $20 \times 150 = 3000$NOK with an investment of NOK1000.
- That is, you will gain 200% of your initial investment.
- However, if you are wrong you will end up with nothing.
- You will have lost 100% of your investment.
- In this strategy one uses a call option to leverage a bet on the behavior of the stock price.

Type of Traders

Type of traders

In the derivatives markets there are three types of traders:

- ***Hedgers***:
  - They use the market to insure themselves against adverse movements of stock prices, currencies, interest rates,...
  - Hedging is an attempt to reduce the exposure to risk that a company already faces.
  - The first example illustrates hedging.
- ***Speculators***:
  - They take a leverage bet on the possible movements of stock prices, currencies, interest rates,...
  - Speculation is needed to make hedging possible.
  - A hedger, wishing to lay off risk, cannot do so unless someone is willing to take it on.
  - The second example illustrates speculation.

Type of traders

- ***Arbitrageurs***:
  - They try to get a risk-less profit by simultaneously entering into transaction in two or more markets/products.
  - By the arbitrage pricing theory the derivatives have a “fair value”.
  - But the market is driven by the law of supply and demand, so the price of a derivative in the market may be different from its fair value.
  - Arbitrageurs eliminate possible mispricings in the market and they are essential in modern financial markets.

Outline of the Course

Course outline

- Time value of the money.
- Basics on arbitrage pricing for forwards and vanilla options.
- Single period financial market models: Pricing and hedging of contingent claims and portfolio optimization.
- Multiperiod financial market models: Pricing and hedging of contingent claims and portfolio optimization.
- Convergence of the binomial model to the Black-Scholes model.
- Risk management with the Black-Scholes model.

Please check the web page of the course for a more detailed syllabus. The main reference for this course:

- S. R. Pliska. Introduction to Mathematical Finance. Discrete Time Models. Blackwell Publishing. (1997)