STK-MAT3700

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Problems

(1) Specify V, V^*, G and G^* in the following models

(a) $K = 3, N = 1, r = 1/9, S_1(0) = 5, S_1(1) = (20/3, 40/9, 30/9)^T$.

(b) $K = 3, N = 2, B(0) = 1, S_1(0) = 5, S_2(0) = 10$,

$$S(1,\Omega) = \begin{pmatrix} 10/9 & 60/9 & 40/3 \\ 10/9 & 60/9 & 80/9 \\ 10/9 & 40/9 & 80/9 \end{pmatrix}.$$

(c) $K = 4, N = 2, B(0) = 1, S_1(0) = 5, S_2(0) = 10$,

$$S(1,\Omega) = \begin{pmatrix} 10/9 & 60/9 & 40/3 \\ 10/9 & 60/9 & 80/9 \\ 10/9 & 40/9 & 80/9 \\ 10/9 & 20/9 & 120/9 \end{pmatrix}.$$

(2) Let $K = 2, N = 1, r = 0$ and $S(0) = 10$, $S(1,\omega_1) = 12$, $S(1,\omega_2) = 10$. Show that there exist the arbitrage opportunities, but the dominant trading stratagy doesn't exist.

(3) Suppose $K = 2; N = 1$ and the interest rate is $r \geq 0$. Also, suppose $S_1(0) = 1, S_1(1,\omega_1) = u, S_1(1,\omega_2) = d$, where $u > d > 0$. For what values of r, u and d there exist a risk neutral probability measure? Give this measure. For the complementary values of these parameters, describe all the arbitrage opportunities.

(4) Let $r = \frac{1}{9}$. Find RNPM, AO.

$S^*(0)$	$S^*(1)$		
	ω_1	ω_2	ω_3
7	8	2	5

(5) Let $r = \frac{1}{9}$. Check whether there exist a risk neutral probability measure. What can you say about the arbitrage opportunities?

n	$S_n(0)$	$S_n(1)$	
		ω_1	ω_2
1	7	12/5	18/5
2	8	24/5	12/5

(6) Suppose the model has next Characteristics: $K = 2, N = 1, r = 1/9, S_1(0) = 5, S(1) = (20/3, 40/9)^T$. Using the direct approach, solve the optimal portfolio problem for the initial wealth $v \geq 0$ and the probability $P(\omega_1) = p$ under the utility functions
 (a) $U(u) = -\exp(-u)$,
 (b) $U(u) = \gamma^{-1}u^{\gamma}$, where $-\infty < \gamma < 1, \gamma \neq 0$.

(7) Suppose $U(u) = \log(u)$. Show that the inverse function $I(i) = i^{-1}$, the optimal Lagrange multiplier $\hat{\lambda} = v^{-1}$, the optimal attainable wealth is $\hat{W} = vL^{-1}B(1)$, and the optimal objective value is $\mathbb{E}\left[U\left(\hat{W}\right)\right] = \log(v) - \mathbb{E}\left[\log\left(LB^{-1}(1)\right)\right]$. Compute these expressions and solve for the optimal trading strategy in the model in exercise 14 with $p = 3/5$.

(8) Suppose $U(u) = \gamma^{-1}u^{\gamma}$, where $-\infty < \gamma < 1, \gamma \neq 0$. Show that the inverse function $I(i) = i^{-\frac{1}{1-\gamma}}$, the optimal Lagrange multiplier

$$\hat{\lambda} = v^{-(1-\gamma)}\left(\mathbb{E}\left[\left(LB^{-1}(1)\right)^{-\frac{\gamma}{1-\gamma}}\right]\right)^{1-\gamma},$$

the optimal attainable wealth

$$\hat{W} = \frac{v\left(LB^{-1}(1)\right)^{-\frac{1}{1-\gamma}}}{\mathbb{E}\left[\left(LB^{-1}(1)\right)^{-\frac{\gamma}{1-\gamma}}\right]},$$

and the optimal objective value $\mathbb{E}\left[U\left(\hat{W}\right)\right] = \lambda v/\gamma$. Compute these expressions and solve for the optimal trading strategy in the model in exercise 1 with $p = 3/5$.

(9) Let $Q = (\frac{1}{8}, \frac{1}{2}, \frac{3}{8})^T$ be the unique martingale measure on this market (you do not have to prove this). Consider the following optimal portfolio problem

$$\max_{H \in H} E\left(U(V(1))\right)$$

$$\text{subject to } V(0) = v,$$

where v is a given non-negative real number, and $U(u) = 2u^{1/2}$. Compute the optimal attainable wealth, the optimal objective value and the optimal trading strategy

References

[1] M. Capi'nski and T. Zastawniak. Mathematics for Finance. An Introduction to Financial Engineering. (2003)

[2] S. R. Pliska. Introduction to Mathematical Finance. Discrete Time Models. Blackwell Publishing. (1997)