Basic Financial Derivatives

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1. Let r be the risk-free rate. a) Prove that $F(t,T)$, the forward price for a stock paying a known dividend D at a future time $0 \leq t_D \leq T$, is given by $F(t,T) = \left(S(t) - De^{-r(t_D - t)}\right) e^{r(T-t)} \qquad 0 \leq t \leq t_D$.

   b) Prove that if the divident is being paid continuously over the period $[0,T]$ at a rate r_D, then the formula is given by $F(t,T) = S(t)\, e^{(r-r_D)(T-t)}, \qquad 0 \leq t \leq T$.

2. Let $V(t)$ be the value at time $t \leq T$ of a forward contract with forward price $F(0,T)$ and delivery time T. Show that $V(t) > (F(t,T) - F(0,T))\, e^{-r(T-t)}$ leads to an arbitrage opportunity.

3. Suppose the interest rate r is constant. Given $S(0)$, find the price of the stock after one day such that the marking to market of futures with delivery in 3 months is zero on that day.

4. Find the stock price on the exercise date for a European put option with strike price NOK360 and exercise date in three months to produce a profit of NOK30 if the option is bought for NOK45, financed by a loan at 12% compounded continuously.

5. European call and put options with strike price NOK240 and exercise date in six months are trading at NOK50.9 and NOK77.8. The price of the underlying stock is NOK203.7 and the interest rate is 7.48%. Find an arbitrage opportunity.

6. Prove by a rigorous arbitrage argument that a) $0 \leq C^E \leq C^A$, b) $0 \leq P^E \leq P^A$.

7. Recall the non-negative part function $(x)^+ = \max(x,0)$. Let $0 \leq K_1 < K_2$, $\alpha \in (0,1)$ and $K = \alpha K_1 + (1-\alpha) K_2$. Prove that
$$(x-K)^+ \leq \alpha (x-K_1)^+ + (1-\alpha)(x-K_2)^+ .$$

8. Prove that, on a stock paying no dividends, one has that

   a) $\left(S(0) - Ke^{-rT}\right)^+ \leq C^A < S(0)$. b)$(K - S(0))^+ \leq P^A < K$.

9. A European call option with a strike price of NOK50 costs NOK2. A European put option with a strike price of NOK45 costs NOK3. Explain how a strangle can be created from these two options. What is the pattern of profits from the strangle?

10. What trading position is created from a long strangle and a short straddle when both have the same time to maturity? Assume that the strike price in the straddle is halfway between the two strike prices of the strangle.

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