STK-MAT3700

O.A. TYMOSHENKO

Mandatory assignment

You can typing the solution directly on a computer (for instance with LATEX). The assignment must be submitted as a single PDF file. Scanned pages must be clearly legible. The submission must contain your name, course and assignment number. It is expected that you give a clear presentation with all necessary explanations. Remember to include all relevant plots and figures. Students who fail the assignment, but have made a genuine effort at solving the exercises, are given a second attempt at revising their answers. All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, we may request that you give an oral account.

To pass the assignment you need a score of at least 60p.

Problem 1.

(1) (5p.)How long will it take for a sum of NOK 8000 attracting simple interest to became NOK 8300 if the rate of interest is 9% per year? Also compute the return on this investment.

(2) (5p.) Suppouse that NOK 24500 is deposited at 5.25% compounded continuously. Find the compound amount the interest earned after 6.5 years, effective rate. Find the time required for the original NOK 24500 to grow to NOK 100000.

(3) (5p.) A loan of NOK 25000 at a rate of 61.5% is paid off in ten years, by paying ten equal installments at the end of every year. How much is each installment?

(4) (5p.) Find the present value of an annuity-due of NOK 20000 per quarter for 2 years, if interest is compounded monthly at the nominal rate of 6%.

(5) (5p.) The cash flows received for every NOK10000 of the five-year corporate bond that pays annual coupon payments of 6%

represented on the time-line (see. Figure 1.) An investor requires an interest rate of 5 % from this bond. What is the value of this bond?

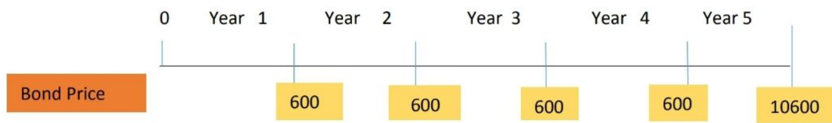

FIGURE 1. The cash flows of bond

(6) (5p.) Find the return on a 65-day investment in zero-coupon bonds if $B(0,1) = 0.8$.

(7) (10p.) Let C^E, P^E, C^A, and P^A denote prices of a European call option, a European put option, an American call option and an American put option, respectively. All of them with expiry time T and the same strike price K. Let $r \geq 0$ be the continuously compounded interest rate. Show that:

1. If
$$C^E - P^E - S(0) + Ke^{-rT} < 0,$$
then you can make a sure risk-less profit.

2. If
$$C^A - P^A - S(0) + Ke^{-rT} > 0,$$
then you can make a sure risk-less profit.

(8) (10p.) A call option with strike price of NOK60 costs NOK6. A put option with the same strike and expiration date costs NOK4. Construct a table that shows the profit from a straddle. For what range of stock prices would the straddle lead to a loss? You may assume that the interest rate is zero.

Problem 2.

(1) (5p.) The price of the stock today is NOK30. Using Table 1 find the price of the shares and draw a graph of changes in the price of the shares.

Table 1.

	$K(1)$	$K(2)$	$K(3)$
ω_1	**5%**	**5%**	**-10%**
ω_2	**-5%**	**-10%**	**5%**
ω_3	**5%**	**-5%**	**5%**

Find the coefficients of return $K(0;2)$, $K(0;3)$ and compare the results with the values of $K(1)+K(2)$ and $K(1)+K(2)+K(3)$. Make a conclusion.

(2) (5p.) The price of one bond at time t denoted by $B(t)$. The price of one share at time t denoted by $S(t)$. Let $B(0)=80$ NOK, $B(1)=100$ NOK, $S(0)=30$ NOK and let $S(1)=40$ NOK with probability 0.6, 25 with probability 0.4. For a portfolio with $x=10$ shares and $y=15$ bonds calculate $V(0)$, $V(1)$ and K_V .

Problem 3.

Consider a single-period market consisting of a probability space $\Omega=\{\omega_1,\omega_2,\omega_3\}$, a probability measure $P(\omega)>0, \omega\in\Omega$, a bank account with $B(0)=1$, and $r=\frac{1}{9}$, and two risky assets, denoted by $S_1=\{S_1(t)\}_{t=0,1}$ and $S_2=\{S_2(t)\}_{t=0,1}$. (See Table 2.)

Table 2.

n	$S_n(0)$	$S_n(1)$		
		ω_1	ω_2	ω_3
1	5	20/3	20/3	40/9
2	10	40/3	80/9	80/9

(1) (10p.) Find $S_n^*(0)$, $S_n^*(1,\omega_i)$, for $n=1,2$ and $i=1,2,3$. Express magnitudes V^* through the strategy $H=(H_0,H_1,H_2)$ for $\omega_i, i=1,2,3$.

(2) (10p.) Define dominant trading strategy and arbitrage opportunity. How are these concepts related?

(3) (10p.) Define linear pricing measure and risk neutral pricing measure. How are these concepts related?

Problem 4.

Consider a single-period market consisting of a probability space $\Omega=\{\omega_1,\omega_2,\omega_3\}$, a probability measure $P(\omega)>0, \omega\in\Omega$, a bank account with $B(0)=1$, and $B(1)=1+r$, where $r\geq 0$ is a given interest rate, and one risky asset, denoted by $S_1=\{S_1(t)\}_{t=0,1}$,

$$S_1(0)=3, \qquad S_1(1,\omega)=\begin{cases} 4 & \text{if} \quad \omega=\omega_1 \\ 3 & \text{if} \quad \omega=\omega_2 \\ 2 & \text{if} \quad \omega=\omega_3 \end{cases}.$$

(1) (10p.) Determine the risk-neutral probability measures. Is the market free of arbitrage? Discuss the result in terms of the possible values of r.

(2) (10p.) What is the definition of a complete market? Is the market complete? Determine (characterize) the attainable claims. Discuss the result in terms of the possible values of r.

(3) (10p.) Set $r = 1/6$. Consider the contingent claim $X = (4, 7/2, 4)^T$. Determine the arbitrage-free prices of X.

(4) (10p.) Set $r = 1/6$. Assume that in the market is introduced a new risky asset S_2 with $S_2(0) = \frac{6}{7}$. Give conditions on $S_2(1) = (S_2(1, \omega_1), S_2(1, \omega_2), S_2(1, \omega_3))^T$ such that the extended market is complete. Check if $S_2(1) = (3/2, 1/3, 1/4)^T$ completes the market and, if this is the case, give the unique risk neutral measure.

References

[1] M. Capi'nski and T. Zastawniak. Mathematics for Finance. An Introduction to Financial Engineering. (2003)

[2] J. C. Hull. Fundamentals of Futures and Options Markets. Pearson. (2017)

[3] Karatzas, I. and Shreve, S.E. *Methods of Mathematical Finance*,Springer-Verlag, Heidelberg, 1997.

[4] S. R. Pliska. Introduction to Mathematical Finance. Discrete Time Models. Blackwell Publishing. (1997)