EXERCISES THAT WILL BE PART OF THE MANDATORY ASSIGNMENT

STK-MAT3700/4700 – AUTUMN 2023

The exercises below will be part of the mandatory assignment. An additional task will be added in the final mandatory assignment and officially posted later. In your mandatory assignment, you need to report how you came up with the answers to the various questions and the software you used and/or developed for this. You are free to choose your software and/or programming language. The date for handing in the mandatory assignments is October 12, as posted on the webpage of the course.

Exercise 1. In this exercise you are supposed to fit the normal distribution to the returns of asset prices. Download time series of asset prices for 5 different companies, both daily closing and weekly closing prices (for example, use Yahoo Finance for this).

    a) Compute and plot the time series of the daily and weekly returns for the 5 companies.

    b) Estimate the mean and volatility of the returns, and report these numbers.

    c) Plot the empirical densities of the returns together with the fitted normal. Make a critical assessment of the normal distribution hypothesis on returns.

Exercise 2. In this task you are asked to analyse the efficient portfolio frontier based on the 5 *weekly* time series of asset prices that you studied in the previous exercise. You are going to study the efficient portfolio frontier in the three cases when you have (i) three assets to invest in, (ii) four assets to invest in, and (iii) five assets to invest in. In your study, use numerical methods to calculate the inverse of matrices and other matrix operations.

    a) Select 3 of the assets, and estimate the variance-covariance matrix for the returns of these three (which is a 3×3-matrix). Next, add one asset, and estimate the variance-covariance matrix for these 4 assets (which is a 4×4-matrix), and finally estimate the variance-covariance matrix for the returns of all 5 assets (which is a 5×5-matrix).

    b) Calculate the minimal-variance portfolio expected return r_m, volatility σ_m and allocation $\underline{x}_m^*$ in the three different cases. Discuss your findings.

    c) Plot the efficient portfolio frontier for all the three cases. Discuss your findings.

    d) Consider the case of five assets. Suppose the interest rate is $r_0 = r_m/2$, i.e., half of the expected return of the minimum-variance portfolio. Find the expected return r_T, the volatility σ_T and the allocation $\underline{x}_T^*$ of the tangent portfolio. What is the range of portfolio risk and expected returns you can achieve by mixing optimally a bank investment with a Markowitz portfolio?