20th September, 2023

STK-MAT3700/4700

Mandatory assignment 1 of 1

Submission deadline

Thursday 12th October 2023, 14:30 in Canvas (canvas.uio.no).

Instructions

Note that you have **one attempt** to pass the assignment. This means that there are no second attempts.

You can choose between scanning handwritten notes or typing the solution directly on a computer (for instance with LaTeX). The assignment must be submitted as a single PDF file. Scanned pages must be clearly legible. The submission must contain your name, course and assignment number.

It is expected that you give a clear presentation with all necessary explanations. Remember to include all relevant plots and figures. All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, we may request that you give an oral account.

In exercises where you are asked to write a computer program, you need to hand in the code along with the rest of the assignment. It is important that the submitted program contains a trial run, so that it is easy to see the result of the code.

Application for postponed delivery

If you need to apply for a postponement of the submission deadline due to illness or other reasons, you have to contact the Student Administration at the Department of Mathematics (e-mail: studieinfo@math.uio.no) no later than the same day as the deadline.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination.

Complete guidelines about delivery of mandatory assignments:

uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

GOOD LUCK!

In your mandatory assignment, you need to report how you came up with the answers to the various questions and the software you used and/or developed for this. You are free to choose your software and/or programming language.

Problem 1. In this exercise you are supposed to fit the normal distribution to the returns of asset prices. Download time series of asset prices for 5 different companies, both daily closing and weekly closing prices (for example, use Yahoo Finance for this).

a) Compute and plot the time series of the daily and weekly returns for the 5 companies.

b) Estimate the mean and volatility of the returns, and report these numbers.

c) Plot the empirical densities of the returns together with the fitted normal. Make a critical assessment of the normal distribution hypothesis on returns.

Problem 2. In this task you are asked to analyse the efficient portfolio frontier based on the 5 *weekly* time series of asset prices that you studied in the previous exercise. You are going to study the efficient portfolio frontier in the three cases when you have (i) three assets to invest in, (ii) four assets to invest in, and (iii) five assets to invest in. In your study, use numerical methods to calculate the inverse of matrices and other matrix operations.

a) Select 3 of the assets, and estimate the variance-covariance matrix for the returns of these three (which is a 3×3-matrix). Next, add one asset, and estimate the variance-covariance matrix for these 4 assets (which is a 4×4-matrix), and finally estimate the variance-covariance matrix for the returns of all 5 assets (which is a 5×5-matrix).

b) Calculate the minimal-variance portfolio expected return r_m, volatility σ_m and allocation $\underline{x}_m^*$ in the three different cases. Discuss your findings.

c) Plot the efficient portfolio frontier for all the three cases. Discuss your findings.

d) Consider the case of five assets. Suppose the interest rate is $r_0 = r_m/2$, i.e., half of the expected return of the minimum-variance portfolio. Find the expected return r_T, the volatility σ_T and the allocation $\underline{x}_T^*$ of the tangent portfolio. What is the range of portfolio risk and expected returns you can achieve by mixing optimally a bank investment with a Markowitz portfolio?

Problem 3. In this exercise you are going to develop a Black & Scholes pricing calculator for call options, and use this to compute the *implied volatility.*

a) Implement the Black & Scholes formula for the price of call options. Calculate the price of 6 call options on an underlying of your choice. You choose strike prices K equal to the current stock price of the underlying, and strikes being $\pm 10\%$ away from current stock price. The exercise times T you choose to be in one month, and in 3 months.

In this exercise, we measure time in *years*, so that 1 month is equal to $1/12$ years, say. You reason yourself what the risk-free interest rate should be, but you should argue for your choice. For the volatility σ, you estimate this from daily stock price data that you download from yahoo-finance (you can use what you have found in the exercises above, if you like). This gives you the *daily* volatility. In your Black & Scholes calculator, you must *annualise* this volatility by scaling your daily estimated value by $\sqrt{250}$. 250 is a usual representation of the number of trading days in a year.

b) Download a set of call option prices for different strike prices but with the same exercise time. Such prices can be found on yahoo-finance, under "options" for the asset of your choice. You choose the exercise time and the strikes, but, you must use strikes above and below (and possibly at) the current value of the stock. Plot the option prices as a function of strike along with the prices that you get from the Black & Scholes formula. Remember to find out how many days there are to your chosen exercise time, and convert this to years assuming 250 trading days in a year.

c) Find the implied volatility from each of the options that you analysed in **b)** above. Plot the implied volatility as a function of strike.

Here you can use a ready-made routine if you like. If not, you can implement a solver to find the implied volatility. Use the following idea: If you want to find the x such that $f(x) = 0$ for a function f, do the following iteration. Find two values x_0 and x_1 such that $f(x_0) < 0$ and $f(x_1) > 0$ (or opposite!!). We know that the desired x is in the interval $[x_0, x_1]$. Then define x_m to be the middle point in the interval $[x_0, x_1]$. Calculate $f(x_m)$, and if $f(x_m) > 0$, then choose the interval $[x_0, x_m]$ (since the desired x must be in this interval), or, if $f(x_m) < 0$, then choose the interval $[x_m, x_1]$ (since in this case the desired x must be here). Let us assume that we ended up choosing $[x_m, x_1]$. Relabel this to $[x_0, x_1]$, and find the middle point again x_m. Check the sign of $f(x_m)$, and choose a new interval according to whether the desired zero is in $[x_0, x_m]$ or $[x_m, x_1]$. Continue until the interval becomes so small that you can tell what the zero is with at least two decimals of accuracy (check interval distance, and continue halfing the interval in the above manner until the distance is less than 0.005).