STK-MAT3710: Extra problems to Chapter 2

Problem 1:

a) Show that

$$(-\infty, x) = \bigcup_{n\in\mathbb{N}} (-\infty, x - \frac{1}{n}]$$

$$(-\infty, x] = \bigcap_{n\in\mathbb{N}} (-\infty, x + \frac{1}{n})$$

b) Assume that $(\Omega, \mathcal{F}, P)$ is a probability space. Prove that the following are equivalent for a function $X\colon \Omega \to \mathbb{R}$:

(i) X is a random variable.

(ii) $\{\omega\colon X(\omega) < x\} \in \mathcal{F}$ for all $x \in \mathbb{R}$.

(iii) $\{\omega\colon X(\omega) > x\} \in \mathcal{F}$ for all $x \in \mathbb{R}$.

(ii) $\{\omega\colon X(\omega) \geq x\} \in \mathcal{F}$ for all $x \in \mathbb{R}$.

Problem 2: Assume that $(\Omega, \mathcal{F}, P)$ is a probability space and that $X\colon \Omega \to \mathbb{R}$ is a random variable:

a) Show that $\{\omega\colon X(\omega) = x\} \in \mathcal{F}$ for all $x \in \mathbb{R}$.

b) Show that the function

$$Y(\omega) = \begin{cases} \frac{1}{X(\omega)} & \text{if } X(\omega) \neq 0 \\ 0 & \text{if } X(\omega) = 0 \end{cases}$$

is a random variable.