STK-MAT3710 Extra Problem

Problem: Assume that X, Y are two sets and that $f\colon X \to Y$ is a function. Recall that if $B \subseteq Y$, the *inverse image of* B *under* f is defined by

$$f^{-1}(B) = \{x \in X \colon f(x) \in B\}$$

a) Show that $f^{-1}(\emptyset) = \emptyset$ and $f^{-1}(Y) = X$.

b) Prove that $f^{-1}(B^c) = \big(f^{-1}(B)\big)^c$.

c) Assume that $\{B_i\}_{i\in I}$ is a family of subsets of Y. Prove that

$$f^{-1}\left(\bigcup_{i\in I} B_i\right) = \bigcup_{i\in I} f^{-1}(B_i) \quad \text{and} \quad f^{-1}\left(\bigcap_{i\in I} B_i\right) = \bigcap_{i\in I} f^{-1}(B_i)$$