Exercises for September 7th

Textbook, pages 20-22: 1.26, 1.32, 1.33, 1.34, 1.37, 1.38, 1.42, 1.43

1.26: Roll two dice

Conditional prob. that one die is 4 given the sum is seven

A = one of the dice show 4

B = sum is seven

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

$P(B)$: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)

$$P(B) = \frac{6}{36} = \frac{1}{6}$$

$P(A\cap B)$ = the prob of one 4 and sum equal 7

$$= \frac{2}{36}$$

$$P(A|B) = \frac{\frac{2}{36}}{\frac{1}{6}} = \frac{2}{36}\cdot\frac{6}{1} = \frac{2}{6} = \frac{1}{3}$$

1.32: Show that if $P(A|C) \geq P(B|C)$ and $P(A|C^c) \geq P(B|C^c)$, then $P(A) \geq P(B)$.

What do the conditions mean:

$$P(A|C) \geq P(B|C) \iff \frac{P(A\cap C)}{P(C)} \geq \frac{P(B\cap C)}{P(C)} \iff P(A\cap C) \geq P(B\cap C)$$

$$P(A|C^c) \geq P(B|C^c) \iff \dots \iff P(A\cap C^c) \geq P(B\cap C^c)$$

Hence

$$P(A) = P(A\cap C) + P(A\cap C^c) \geq P(B\cap C) + P(B\cap C^c) = P(B)$$

1.33: A, B independent with prob p. What is $P(A\cup B)$? Know $P(A\cap B) = P(A)P(B) = p^2$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B) = p + p - p^2 = 2p - p^2$$

1.34: We have $P(B) = \begin{cases} 0 \\ 1 \end{cases}$

Show that A, B are independent for all A.

$P(B) = 0$: We must prove $P(A\cap B) = P(A)P(B)$.

$0 \leq P(A\cap B) \leq P(B) = 0 \Rightarrow P(A\cap B) = 0$

$P(A)P(B) = P(A)\cdot 0 = 0$. Independence!

$P(B) = 1$: $P(A)P(B) = P(A)\cdot 1 = P(A)$.

Also $P(A) = P(A\cap B) + \underbrace{P(A\cap B^c)}_{0} = P(A\cap B)$

Hence $P(A\cap B) = P(A)P(B)$ independence.

1.37 Rolling a die twice

a) First roll is 2, the second roll is odd

Independent

b) First roll is 2, the two rolls are equal.

Independent.

c) $\underbrace{\text{First roll is 2}}_{A}$, $\underbrace{\text{the sum is six}}_{B}$.

$P(A) = \frac{1}{6}$ $P(A|B) = \frac{1}{5}$ not independence

unequal

B: (1,5), (2,4), (3,3), (4,2), (5,1)

1.38: Best of seven = first to four wins

Compare: 7 games or 6 more likely

Trick: After 5 games: $\left.\begin{matrix} 3 \\ 2 \end{matrix}\right\}$ 1/2 $\binom{4}{2}$ = 6 games; 1/2 $\binom{3}{3}$ = 7 games

Prob: 1/2 for each.

1.42: Families with two children

$\cancel{(B,B)}$, (B,G) 1/4, (G,B) 1/4, (G,G) 1/2

a) A family is picked at random, and at least one child is a girl. What is the prob. that the other one is a boy? 2/3

b) Suppose oldest child is a girl. What is the prob that the other one is a boy? 1/2

c) Pick a girl at random. What is the prob that her sibling is a boy? 1/2.

1.43: A is independent of itself:

$$P(A) = P(\underbrace{A\cap A}_{A}) = P(A)P(A) = P(A)^2$$

$$x = x^2$$

$P(A) = 0$ or $P(A) = 1$