Exercises September 7th

Inverse images:

$$f: X \to Y$$

If $B \subseteq Y$, then the inverse image $f^{-1}(B)$ is defined by

$$f^{-1}(B) = \{x \in X : f(x) \in B\}$$

a) Show that $f^{-1}(\emptyset) = \emptyset$, $f^{-1}(Y) = X$.

$$f^{-1}(\emptyset) = \{x \in X : f(x) \in \emptyset\} = \emptyset$$

$$f^{-1}(Y) = \{x \in X : f(x) \in Y\} = X.$$

b) Show that $f^{-1}(B^c) = (f^{-1}(B))^c$

$$x \in f^{-1}(B^c) \Leftrightarrow f(x) \in B^c \Leftrightarrow f(x) \notin B$$

$$\Leftrightarrow x \notin f^{-1}(B) \Leftrightarrow x \in (f^{-1}(B))^c$$

c) Show that

$$f^{-1}\left(\bigcup_{i \in I} B_i\right) = \bigcup_{i \in I} f^{-1}(B_i)$$

$$x \in f^{-1}\left(\bigcup_{i \in I} B_i\right) \Leftrightarrow f(x) \in \bigcup_{i \in I} B_i$$

$\Leftrightarrow f(x) \in B_i$ for at least one $i \in I$

$\Leftrightarrow x \in f^{-1}(B_i)$ —— " ——

$$\Leftrightarrow x \in \bigcup_{i \in I} f^{-1}(B_i)$$

Show that

$$f^{-1}\left(\bigcap_{i \in I} B_i\right) = \bigcap_{i \in I} f^{-1}(B_i)$$

$$x \in f^{-1}\left(\bigcap_{i \in I} B_i\right) \Leftrightarrow f(x) \in \bigcap_{i \in I} B_i$$

$\Leftrightarrow f(x) \in B_i$ for all i

$\Leftrightarrow x \in f^{-1}(B_i)$ for all i

$$\Leftrightarrow x \in \bigcap_{i \in I} f^{-1}(B_i)$$