

Exercises September 7th

$\frac{1}{X}$ is a random variable

a) Show that

$$\{\omega: X(\omega) = x\} \in \mathcal{F}$$

Note that:

$$\{\omega: X(\omega) = x\} = \{\omega: X(\omega) \leq x\} \cap \{\omega: X(\omega) \geq x\} \\ \in \mathcal{F}$$

b) Define

$$Y(\omega) = \begin{cases} \frac{1}{X(\omega)} & \text{if } X(\omega) \neq 0 \\ 0 & \text{if } X(\omega) = 0 \end{cases}$$

Show that Y is a random variable, i.e.

$$\{\omega: Y(\omega) \leq \alpha\} \in \mathcal{F} \text{ for all } \alpha.$$

Assume $\alpha < 0$:

$$\begin{aligned} \{\omega: Y(\omega) \leq \alpha\} &= \{\omega: X(\omega) < 0\} \cap \\ &\quad \{\omega: \frac{1}{X(\omega)} \leq \alpha\} \quad \frac{X(\omega)}{\alpha} \\ &= \underbrace{\{\omega: X(\omega) < 0\}}_{\text{meas}} \cap \underbrace{\{\omega: \frac{1}{\alpha} \leq X(\omega)\}}_{\text{meas}} \in \mathcal{F} \end{aligned}$$

Assume $\alpha < 0$:

$$\{\omega: Y(\omega) \leq 0\} = \{\omega: X(\omega) \leq 0\} \in \mathcal{F}$$

Assume $\alpha > 0$:

$$\{\omega: Y(\omega) \leq \alpha\} = \{\omega: X(\omega) \leq 0\}$$

$$\cup \underbrace{\{\omega: X(\omega) \geq \frac{1}{\alpha}\}}_{\mathcal{F}} \in \mathcal{F}$$