

4.79 Servings of fruits and vegetables. The following table gives the distribution of the number of servings of fruits and vegetables consumed per day in a population.

Number of servings X	0	1	2	3	4	5
Probability	0.3	0.1	0.1	0.2	0.2	0.1

Find the mean for this random variable.

4.80 Mean of the distribution for the number of aces. In Exercise 4.58 (page 262) you examined the probability distribution for the number of aces when you are dealt two cards in the game of Texas hold 'em. Let X represent the number of aces in a randomly selected deal of two cards in this game. Here is the probability distribution for the random variable X :

Value of X	0	1	2
Probability	0.8507	0.1448	0.0045

Find μ_X , the mean of the probability distribution of X .

4.88 What happens when the correlation is 1? We know that variances add if the random variables involved are uncorrelated ($\rho = 0$), but not otherwise. The opposite extreme is perfect positive correlation ($\rho = 1$). Show by using the general addition rule for variances that in this case the standard deviations add. That is, $\sigma_{X+Y} = \sigma_X + \sigma_Y$ if $\rho_{XY} = 1$.

4.91 Fire insurance. An insurance company looks at the records for millions of homeowners and sees that the mean loss from fire in a year is $\mu = \$300$ per person. (Most of us have no loss, but a few lose their homes. The \$300 is the average loss.) The company plans to sell fire insurance for \$300 plus enough to cover its costs and profit. Explain clearly why it would be stupid to sell only 10 policies. Then explain why selling thousands of such policies is a safe business.

4.92 Mean and standard deviation for 10 and for 12 policies. In fact, the insurance company sees that in the entire population of homeowners, the mean loss from fire is $\mu = \$300$ and the standard deviation of the loss is $\sigma = \$400$. What are the mean and standard deviation of the average loss for 10 policies? (Losses on separate policies are independent.) What are the mean and standard deviation of the average loss for 12 policies?

4.93 Life insurance. Assume that a 25-year-old man has these probabilities of dying during the next five years:

Probability	0.00039	0.00044	0.00051	0.00057	0.00060
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(a) What is the probability that the man does not die in the next five years?

(b) An online insurance site offers a term insurance policy that will pay \$100,000 if a 25-year-old man dies within the next five years. The cost is \$175 per year. So the insurance company will take in \$875 from this policy if the man does not die within five years. If he does die, the company must pay \$100,000. Its loss depends on how many premiums the man paid, as follows:

Age at death	25	26	27	28	29
Loss	\$99,825	\$99,650	\$99,475	\$99,300	\$99,125

What is the insurance company's mean cash intake from such policies?

4.111 Exercise and sleep. Suppose that 40% of adults get enough sleep, 46% get enough exercise, and 24% do both. Find the probabilities of the following events:

- enough sleep and not enough exercise
- not enough sleep and enough exercise
- not enough sleep and not enough exercise
- For each of parts (a), (b), and (c), state the rule that you used to find your answer.

4.112 Exercise and sleep. Refer to the previous exercise. Draw a Venn diagram showing the probabilities for exercise and sleep.

4.115 Attendance at two-year and four-year colleges. In a large national population of college students, 61% attend four-year institutions and the rest attend two-year institutions. Males make up 44% of the students in the four-year institutions and 41% of the students in the two-year institutions.

(a) Find the four probabilities for each combination of gender and type of institution in the following table. Be sure that your probabilities sum to 1.

	Men	Women
Four-year institution		
Two-year institution		

(b) Consider randomly selecting a female student from this population. What is the probability that she attends a four-year institution?

4.116 Draw a tree diagram. Refer to the previous exercise. Draw a tree diagram to illustrate the probabilities in a situation where you first identify the type of institution attended and then identify the gender of the student.

4.118 Education and income. Call a household prosperous if its income exceeds \$100,000. Call the household educated if the householder completed college. Select an American household at random, and let A be the event that the selected household is prosperous and B the event

that it is educated. According to the Current Population Survey, $P(A) = 0.138$, $P(B) = 0.261$, and the probability that a household is both prosperous and educated is $P(A \text{ and } B) = 0.082$. What is the probability $P(A \text{ or } B)$ that the household selected is either prosperous or educated?

4.119 Find a conditional probability. In the setting of the previous exercise, what is the conditional probability that a household is prosperous, given that it is educated? Explain why your result shows that events A and B are not independent.

4.131 Muscular dystrophy. Muscular dystrophy is an incurable muscle-wasting disease. The most common and serious type, called DMD, is caused by a sex-linked recessive mutation. Specifically, women can be carriers but do not get the disease; a son of a carrier has probability 0.5 of having DMD; a daughter has probability 0.5 of being a carrier. As many as one-third of DMD cases, however, are due to spontaneous mutations in sons of mothers who are not carriers. Toni has one son, who has DMD.

In the absence of other information, the probability is $1/3$ that the son is the victim of a spontaneous mutation and $2/3$ that Toni is a carrier. There is a screening test called the CK test that is positive with probability 0.7 if a woman is a carrier and with probability 0.1 if she is not. Toni's CK test is positive. What is the probability that she is a carrier?