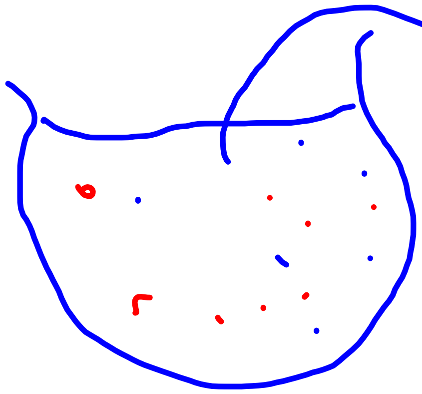
urne med røde & blå baller



20: 5 blå og 15 røde

$p$ ~ Andelen <u>røde</u> baller

$\mathrm{Bin}(n=20, p)$

$X = 15$

$$lik = \underbrace{\binom{20}{15}}_{\text{et tall } K} \cdot p^{15} \cdot (1-p)^5$$

$$\log lik = \log K + 15 \log p + 5 \log(1-p)$$

$$\frac{\partial\, loglik}{\partial p} = 0 + \frac{15}{p} - \frac{5}{1-p}$$

$$\frac{15}{p} = \frac{5}{1-p}$$

$$15(1-p) = 5p$$

$$15 = 20p$$

$$\boxed{\hat{p} = 15/20}$$

$m$ røde baller av $n$, $m \leq n$

$$\underline{\hat{p} = m/n}$$

$X_1, X_2, \dots, X_n \sim N(\mu, \sigma)$
uavh.

$\sigma$ kjent

Vis at $\bar{X}$ er ML-est for $\mu$.

$f(x_i) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left(-\frac{1}{2\sigma^2}(x_i-\mu)^2\right)$

$lik = \prod_{i=1}^{n} f(x_i) = (2\pi\sigma^2)^{-n/2} \cdot \exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2\right)$

$loglik = -\frac{n}{2}\log(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2$

$\frac{\partial loglik}{\partial \mu} = 0 + \frac{1}{2\sigma^2}\underbrace{\sum_{i=1}^{n} 2(x_i-\mu)}$

$\sum_{i=1}^{n}(x_i-\mu) = 0$

$\cancel{n} \cdot \bar{x} - \cancel{n}\mu = 0$

$\mu = \bar{x}$

ML $\hat{\mu} = \bar{X}$