Tilfeldig variabel X

Diskret

Verdier x_i, sannsynlighet p_i

$\mu = \sum x_i p_i$

$Var(X) = \sum (x_i - \mu)^2 \cdot p_i$

Kontinuerlig X

$\mu = \int x f(x)\,dx$, $Var(X) = \int (x-\mu)^2 f(x)\,dx$

$\mu_{\frac{X+Y}{2}} = \mu_{\frac{X}{2}+\frac{Y}{2}} \overset{\text{Regel 2}}{=} \mu_{\frac{X}{2}} + \mu_{\frac{Y}{2}}$

$\overset{\text{Regel 1}}{=} \frac{1}{2}\mu_X + \frac{1}{2}\mu_Y = \frac{\mu_X + \mu_X}{2} //$

$\mu_{\frac{X+Y}{2}} \overset{\text{Regel 1}}{=} \frac{1}{2}\mu_{X+Y} \overset{\text{Regel 2}}{=} \frac{1}{2}(\mu_X + \mu_Y)$

$X_1, X_2, \ldots, X_n$

$\mu_{\frac{X_1+X_2+\ldots+X_n}{n}} \overset{\text{Regel 1}}{=} \frac{1}{n}\mu_{S_n}$

$S_n = X_1 + X_2 + \ldots + X_n$

$a=0, b=\frac{1}{n}$

$S_2 = X_1 + X_2$ Regel 2 $\mu_{S_1} = \mu_{X_1+X_2} = \mu_{X_1} + \mu_{X_2}$

$S_3 = X_1 + X_2 + X_3 = S_2 + X_3$ $\mu_{S_3} = \mu_{(S_2+X_3)} = \mu_{S_2} + \mu_{X_3}$

$X_1 + X_2 + X_3 + X_4 + \ldots + X_n$

S_2

S_3

S_4

$S_n = X_1 + X_2 + X_3 + \ldots + X_n$

$\mu_{S_3} = \mu_{X_1} + \mu_{X_2} + \mu_{X_3}$

$\mu_{S_n} = \mu_{X_1} + \mu_{X_2} + \mu_{X_3} + \ldots + \mu_{X_n}$

3. $\mu_{\frac{X_1+X_2+\ldots+X_n}{n}} = \mu_{\frac{S_n}{n}} \overset{\text{Regel 1}}{=} \frac{1}{n}\mu_{S_n}$

4 $\mu_{S_n} = \mu_{X_1} + \mu_{X_2} + \ldots + \mu_{X_n}$

$\mu_{\bar{X}} = \mu_{\frac{X_1+X_2+\ldots+X_n}{n}} = \frac{\mu_{X_1} + \mu_{X_2} + \ldots + \mu_{X_n}}{n}$

Uavhengige X_1, X_2, X_3

$\sigma^2_{\frac{X_1+X_2+X_3}{3}} \overset{\text{Regel 1}}{=} \left(\frac{1}{3}\right)^2 \cdot (\sigma^2_{X_1} + \sigma^2_{X_2} + \sigma^2_{X_3})$

$= 1/9$

$\sigma^2_{\frac{X_1+X_2+X_3}{3}} \overset{\text{Regel 1}}{=} \frac{1}{9}\sigma^2_{X_1+X_2+X_3} = \frac{\sigma^2_{X_1} + \sigma^2_{X_2} + \sigma^2_{X_3}}{9}$

Regel 2: $\sigma^2_{X_1+X_2} = \sigma^2_{X_1} + \sigma^2_{X_2}$

$\sigma^2_{X_1+X_2+X_3} = \sigma^2_{X_1+X_2} + \sigma^2_{X_3} = \sigma^2_{X_1} + \sigma^2_{X_2} + \sigma^2_{X_3}$

$$\sigma^2_{\frac{X_1+X_2+\ldots+X_n}{n}} = \frac{\sigma^2_{X_1}+\sigma^2_{X_2}+\ldots+\sigma^2_{X_n}}{n^2}$$

$$\sigma_{\frac{X_1+X_2+\ldots+X_n}{n}} = \frac{\sqrt{\sigma^2_{X_1}+\ldots+\sigma^2_{X_n}}}{n}$$

$X_1, X_2 \ldots, X_n$ DERSOM

$\mu_{X_1}=\mu_{X_2}=\ldots=\mu_X$

$\sigma_{X_1}=\sigma_{X_2}=\ldots=\sigma_X$

$$\mu_{S_n/n} = \frac{\mu_{X_1}+\ldots+\mu_{X_n}}{n} = \mu_X$$

$$\sigma_{S_n/n} = \frac{\sqrt{\sigma^2_{X_1}+\ldots+\sigma^2_{X_n}}}{n} = \frac{\sqrt{n\sigma^2_{X_1}}}{n} = \frac{\sqrt{n}}{n}\sigma_{X_1} = \sigma_{X_1}/\sqrt{n}$$