

Eksamen 2021

1a) $Y_1 \sim N(\mu_1, \sigma_1)$

$Y_2 \sim N(\mu_2, \sigma_2)$

i) $Y_1 + Y_2 \sim N(\mu, \sigma)$

normalfordelt

$\mu = \mu_1 + \mu_2$

$\sigma_{Y_1+Y_2}^2 = \sigma_{Y_1}^2 + \sigma_{Y_2}^2 = \sigma_1^2 + \sigma_2^2$

$\sigma = \sigma_{Y_1+Y_2} = \sqrt{\sigma_1^2 + \sigma_2^2}$



B - ønsker å kutte

M - merker av

Z - der jeg faktisk kutter

$M = B + \varepsilon_1 \quad \varepsilon_1 \sim N(0, \sigma_1)$

$Z = M + \varepsilon_2 \quad \varepsilon_2 \sim N(0, \sigma_2)$

$= B + \varepsilon_1 + \varepsilon_2$

$\sigma_Z = \sigma_{\varepsilon_1 + \varepsilon_2} = \sqrt{\sigma_1^2 + \sigma_2^2}$
 $= \sqrt{2} \cdot 0.05 \text{ cm} = 0.07 \text{ cm}$



1c) i) $P(Z > 80) = 0.5$

$P(Z > 80) = P\left(\tilde{Z} > \frac{80 - 80}{0.07}\right) = P(\tilde{Z} > 0) = 0.5$

ii)

$Z = B + \varepsilon_1 + \varepsilon_2$

$Z \sim N(B, \sqrt{2} \cdot 0.05)$

95% persentil for til Z skal være 80.00 cm

95% persentilen til Z = $B + z_{0.95} \cdot \sigma_Z$

$B + z_{0.95} \cdot 0.07 = 80.00$

$B = 80.00 - z_{0.95} \cdot 0.07 = 80.00 - 1.64 \cdot 0.07$
 $= \underline{\underline{79.9}}$

2a) i) Binomisk fordeling

- Fast ant. rep. $n=100$
- I hvert forsøk: to mulige utfall
 $X_i > Y_i$ eller $Y_i > X_i$
Suksess
- Fast suksess-sannsynlighet $p=0.5$
- Hver av repkopiene er uafhængige
Alle 200 var er uafh. $\Rightarrow X_i$ og Y_i uafh. i hvert forsøk
thet per er uafh. or berand

ii) N væe ant. per der
 $X_i > Y_i$
 $N \sim \text{Bin}(100, 0.5)$
 $\mu_N = n \cdot p = 100 \cdot 0.5 = 50$
 iii) $\sigma_N = \sqrt{np(1-p)} = 5$
 $P(N < 40) \approx P(N \leq 39) \approx$
 $P(\tilde{N} < 39.5) = P(Z < \frac{39.5 - 50}{5})$
normalfordelt
 $N(50, 5)$
std. normalfordelt
 $= P(Z < -2.1) = 0.0179$

2b) $X_i \sim N(\mu_x, \sigma)$
 $Y_i \sim N(\mu_y, \sigma)$
 $X_i - Y_i \sim N(\mu, \sigma_2)$
normalfordelt
 $\mu = \mu_{X_i - Y_i} = \mu_x - \mu_y$
 $\sigma_2^2 = \sigma_x^2 + \sigma_y^2 = 2\sigma^2$
 $\sigma_2 = \sqrt{2} \cdot \sigma$

ii) $\bar{X} - \bar{Y} \sim N(\mu_{\bar{X}-\bar{Y}}, \sigma_{\bar{X}-\bar{Y}})$
normalfordeling
 $\mu_{\bar{X}-\bar{Y}} = \mu_{\bar{X}} - \mu_{\bar{Y}} = \mu_x - \mu_y$
 $\sigma_{\bar{X}-\bar{Y}}^2 = \sigma_{\bar{X}}^2 + \sigma_{\bar{Y}}^2 = \frac{\sigma^2}{n} + \frac{\sigma^2}{n} = 2 \frac{\sigma^2}{n}$
 $\sigma_{\bar{X}-\bar{Y}} = \sqrt{2} \cdot \sigma / \sqrt{n} = \frac{\sqrt{2} \cdot \sigma}{\sqrt{n}}$

2c) Stat. signifikans-test
 Velges stat. sign. $\alpha = 0.05$
 $H_0: \mu_x = \mu_y \Leftrightarrow \mu_x - \mu_y = 0$
 $H_2: \mu_x \neq \mu_y$
 Std. testobservator $T = \frac{\bar{X} - \bar{Y} - (\mu_x - \mu_y)}{S_{\bar{X}-\bar{Y}}}$
 $= \frac{10.02 - 10.98}{4.73/\sqrt{100}}$
 $= -2.03$
 $df: n-1 = 99$
 $P\text{-verdi} = 2 \cdot P(T > 2.03)$
 < 0.025
 $< 2 \cdot 0.025 = 0.05$
 Vi har obs. p-verdi $< \alpha$
 Forkaster H_0 til fordel for H_2
 og konkluderer at forv. μ_x og μ_y er ulike.

32)

$$i: y_i = \beta_0 + \beta_1 \cdot X_i + \varepsilon_i$$

$$\varepsilon_i \sim N(0, \sigma)$$

uavh.

ii)

3/p) (i) 95% KI for β_1

$$b_1 \pm t^* \cdot SE(b_1)$$

$$1.27 \pm 1.984 \cdot 0.038$$

$$[1.194, 1.345]$$

(ii)

(iii)

predict(fit, newdata =
data.frame(fit.novle=104),
interval = 'confidence',
level = 0.95)

iii) predict(fit, newdata = data.frame(fit.novle=104),
interval = 'predict', level = 0.95)

iv) Se formuleringer i
løsningsforslaget.

Stikkord: mulige verdier i samsvar
med dataene

- hva estimeres vi (β_1, μ_{y_i}, y_i)
- Konfidensnivå: [95%] av tilfelle
metoden blir brukt, vi....

Oppg 4

a) Logistisk regresjonsmodell

$$P_i = P(y_i = 1 | x_{i1}, x_{i2})$$

Snø til jul

$$\log \frac{P_i}{1 - P_i} = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2}$$

b) 95% KI for β_2
den naturlige logaritmen av OR
for snø til jul på hytta snø hjemme

$$b_1 \pm Z^* \cdot SE(b_1)$$

$$0.76152 \pm 1.96 \cdot 0.84862$$

$$[-0.902, 2.425]$$

95% KI for OR

$$[e^{-0.902}, e^{2.425}]$$

$$= [0.41, 11.30]$$

Table entry for z is the area under the standard Normal curve to the left of z .

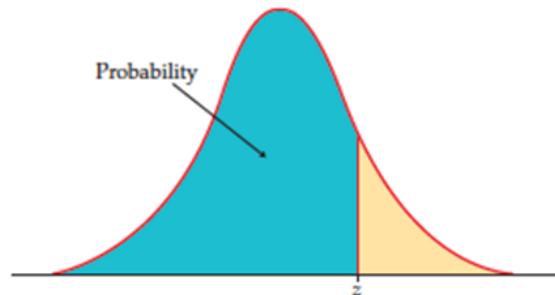


TABLE A Standard Normal Probabilities (continued)										
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table entry for z is the area under the standard Normal curve to the left of z .

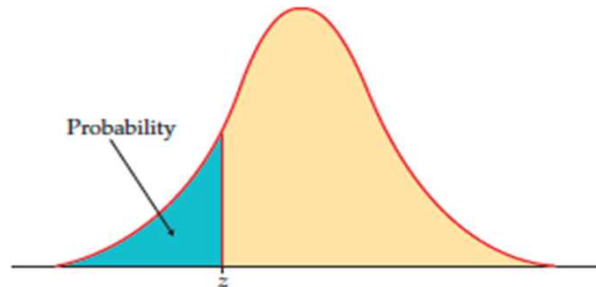


TABLE A Standard Normal Probabilities										
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table entry for p and C is the critical value t^* with probability p lying to its right and probability C lying between $-t^*$ and t^* .

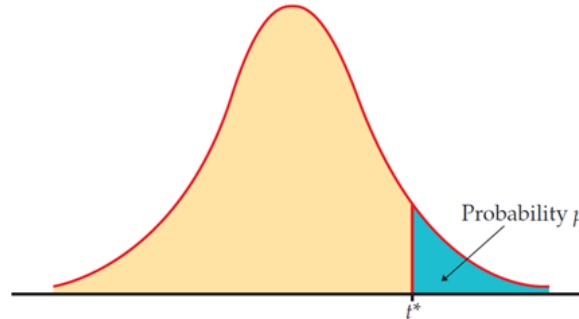


TABLE D

t distribution critical values

df	Upper-tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											