Konfidensintervall

kap 6

Et utvalg: $X_1, \dots, X_n \sim N(\mu, \sigma)$, σ kjent

KI for μ: $\boxed{\bar{x} \pm z^* \cdot \sigma_{\bar{x}}}$, $\sigma_{\bar{x}} = \sigma/\sqrt{n}$

Et utvalg $X_1, \dots, X_n \sim N(\mu, \sigma)$, σ ukjent kap 7

best. empirisk std.av

KI for μ $\bar{x} \pm t^* \cdot s/\sqrt{n}$

$$df = n-1$$

Matchede par

$X_1, \dots, X_n$

$Y_1, \dots, Y_n$

Mål er å undersøke eller ressonnere rundt $\mu_X - \mu_Y$.

$Z_i = Y_i - X_i \quad i = 1, \dots, n$

Analysere differansene Z_i som ett utvalg med n obs ("ett-utvalgs t-test")

KI for $\mu_Z = \mu_Y - \mu_X$

$$\bar{z} \pm t^* \frac{s_z}{\sqrt{n}}$$

$$df = n-1$$

$$\sigma^2_{\bar{X}-\bar{Y}} = \sigma^2_{\bar{X}+(-\bar{Y})} = \sigma^2_{\bar{X}} + \sigma^2_{\bar{Y}} = \frac{\sigma_X^2}{n_X} + \frac{\sigma_Y^2}{n_Y}$$

To utvalg, σ_1, σ_2 kjent

UAVH.

1. utvalg $X_1, \ldots, X_{n_1} \sim N(\mu_1, \sigma_1)$

2. utvalg $Y_1, \ldots, Y_{n_2} \sim N(\mu_2, \sigma_2)$

Alt A: σ_1, σ_2 kjente OG like $\sigma_1 = \sigma_2$

$$\sigma_{\overline{X}-\overline{Y}} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \overset{\sigma_1=\sigma_2}{=} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_1^2}{n_2}}$$

$$= \sigma_1 \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

KI for $\mu_1 - \mu_2$ $\quad (\overline{X} - \overline{Y}) \pm z^* \cdot \sigma_{\overline{X}-\overline{Y}}$

AltB: σ_1, σ_2 kjente og ulike

$$\sigma_{\overline{X}-\overline{Y}} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

KI for $\mu_1 - \mu_2$ $\quad (\overline{X} - \overline{Y}) \pm z^* \cdot \sigma_{\overline{X}-\overline{Y}}$

To utvalg σ_1, σ_2 UKJENT og ulike

KI for $\mu_1 - \mu_2$ $(\bar{X} - \bar{Y}) \pm t^* \cdot \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$

$df = \min(n_1 - 1, n_2 - 1)$

σ_1, σ_2 UKJENTE og like

$$S_{pooled}^2 = \frac{(n_1 - 1) S_1^2 + (n_2 - 1) \cdot S_2^2}{n_1 + n_2 - 2}$$

KI for $\mu_1 - \mu_2$ $(\bar{X} - \bar{Y}) \pm t^* S_{pooled} \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

$$df = n_1 + n_2 - 2$$

Eks: −1, −1, 0, 1, 1

5 obs $\bar{X} = 0$, $Var(X) = \frac{4}{5}$, $S_X = \frac{2}{\sqrt{5}}$

6 obs Y $\bar{Y} = \frac{1}{2}$, $S_Y = \frac{3}{\sqrt{5}}$

$$S^2_{pooled} = \frac{4\cdot\left(\frac{2}{\sqrt{5}}\right)^2 + 5\cdot\left(\frac{3}{\sqrt{5}}\right)^2}{9} = \frac{\frac{16}{5} + \frac{45}{5}}{9} = \frac{61}{45}$$

KI for $\mu_1 - \mu_2$: $(\bar{X} - \bar{Y}) \pm t^* \sqrt{\frac{61}{45}} \cdot \sqrt{\frac{1}{5} + \frac{1}{6}}$

$(\bar{X} - \bar{Y}) = -\frac{1}{2}$

t^*: Nivå C, df

Eks: 95%, 9

$-\frac{1}{2} \pm 2.26 \cdot$

Linear regresjon

KI for β_1 $b_1 \pm t^* \cdot SE_{b_1}$

KI for μ_{y_i} for en gitt x_i $\hat{\mu}_{y_i} \pm t^* \cdot SE_{\hat{\mu}_{y_i}}$

Logistisk regresjon

KI for β_1 $b_1 \pm z^* \cdot SE_{b_1}$ Eks $[5, 7]$

KI for OR $e^{b_1 \pm z^* SE_{b_1}}$ $[e^5, e^7]$

assosiert med én enhets økning i X_{i1}

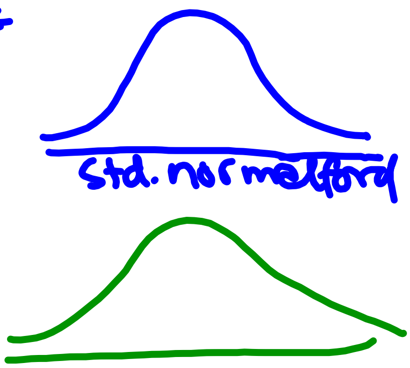
(når de andre forkl. var. holdes fast)

Nivå C konfidensintervall

$0 \leq C \leq 1$

z^*

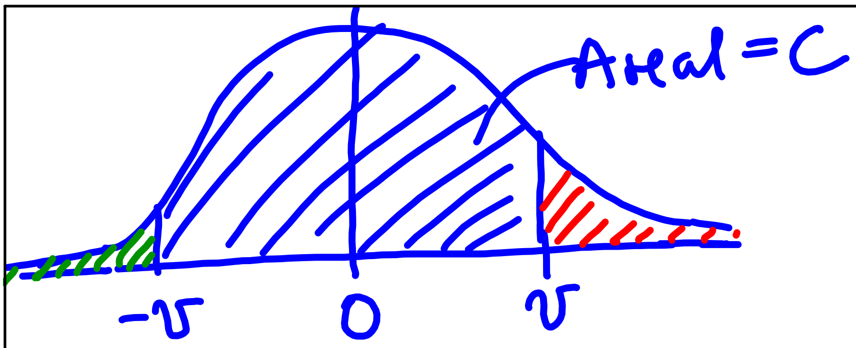
t^*



Klokkeformet

Symmetrisk

Sentrert i 0

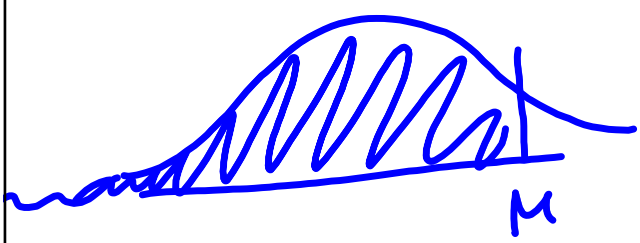


////// = ////// = $\frac{1-C}{2}$

KI "med z^*": Finn v s.a. $\int_{-v}^{v} f(x)\,dx = C$ (f: tetth. norm)

KI "med t^*": Finn v s.a. $\int_{-v}^{v} f(x)\,dx = C$ (f: tetthetsfu. t)

Finn u s.a. $\int_{-\infty}^{u} f(x)\,dx = K$



$K = C + \frac{1-C}{2}$

Table entry for p and C is the critical value t^* with probability p lying to its right and probability C lying between $-t^*$ and t^*.

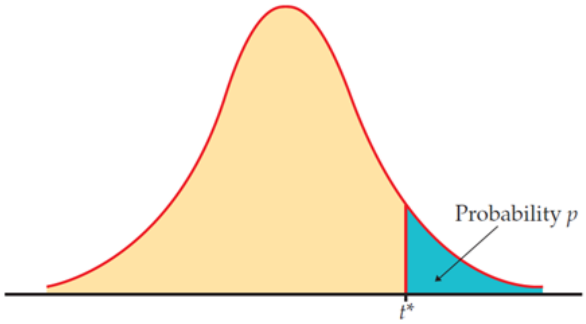


TABLE D

t distribution critical values

	Upper-tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											