4.46 Discrete or continuous. In each of the following situations, decide whether the random variable is discrete or continuous and give a reason for your answer.

(a) Your web page has five different links, and a user can click on one of the links or can leave the page. You record the length of time that a user spends on the web page before clicking one of the links or leaving the page.

(b) You record the number of hits per day on your web page.

(c) You record the yearly income of a visitor to your web page.

4.54 How many close friends? How many close friends do you have? Suppose that the number of close friends adults claim to have varies from person to person with mean $\mu = 8$ and standard deviation $\sigma = 3$. An opinion poll asks this question of an SRS of 500 adults. We will see in the next chapter that, in this situation, the sample mean response $\bar{x}$ has approximately the Normal distribution with mean 8 and standard deviation 0.1342. What is $P(7.1 \leq \bar{x} \leq 8.1)$, the probability that $\bar{x}$ estimates μ to within ± 0.1?

4.51 Find the probabilities. Let the random variable X be a random number with the uniform density curve in Figure 4.9 (page 229). Find the following probabilities:

(a) $P(X \geq 0.40)$.

(b) $P(X = 0.40)$.

(c) $P(0.40 < X < 1.40)$.

(d) $P(0.22 \leq X \leq 0.25 \text{ or } 0.42 \leq X \leq 0.45)$.

(e) X is not in the interval 0.5 to 0.8.

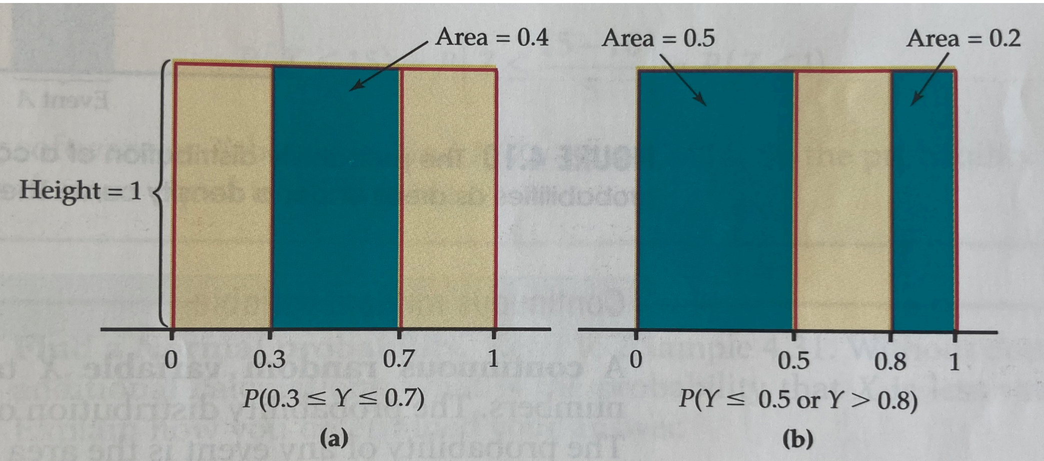

FIGURE 4.9 Assigning probabilities for generating a random number between 0 and 1, Example 4.29. The probability of any interval of numbers is the area above the interval and under the density curve.

Section 4.4 EXERCISES

4.56 Different kinds of means. Explain the difference between the mean of a random variable and the mean of a sample.

4.57 Find the mean of the random variable. A random variable X has the following distribution:

X	−2	−1	0	1
Probability	0.1	0.2	0.4	0.3

Find the mean for this random variable. Show your work.

4.58 Servings of fruits and vegetables. The following table gives the distribution of the number of servings of fruits and vegetables consumed per day in a population:

Number of servings X	0	1	2	3	4	5
Probability	0.4	0.1	0.1	0.2	0.1	0.1

Find the mean for this random variable.

4.59 Explain what happens when the sample size gets large. Consider the following scenarios: (1) You take a sample of two observations on a random variable and compute the sample mean, (2) you take a sample of 100 observations on the same random variable and compute the sample mean, (3) you take a sample of 1000 observations on the same random variable and compute the sample mean. Explain in simple language how close you expect the sample mean to be to the mean of the random variable as you move from Scenario 1 to Scenario 2 to Scenario 3.

4.60 What's wrong? In each of the following scenarios, there is something wrong. Describe what is wrong and give a reason for your answer.

(a) If you toss a fair coin three times and get heads all three times, then the probability of getting a tail on the next toss is much greater than one-half.

(b) If you multiply a random variable by 10, then the mean is multiplied by 10 and the variance is multiplied by 10.

(c) When finding the mean of the sum of two random variables, you need to know the correlation between them.

4.61 Find some means. Suppose that X is a random variable with mean 20 and standard deviation 2. Also suppose that Y is a random variable with mean 40 and standard deviation 7. Assume that the correlation between X and Y is zero. Find the mean of the random variable Z for each of the following cases. Be sure to show your work.

(a) $Z = 25 - 12X$.

(b) $Z = 13X - 8$.

(c) $Z = X + Y$.

(d) $Z = X - Y$.

(e) $Z = -3X + 3Y$.

4.62 Mean of the distribution for the number of aces. In Exercise 4.47 (page 234) you examined the probability distribution for the number of aces when you are dealt two cards in the game Texas hold 'em. Let X represent the number of aces in a randomly selected deal of two cards in this game. Here is the probability distribution for the random variable X:

Value of X	0	1	2
Probability	0.8507	0.1448	0.0045

Find μ_X, the mean of the probability distribution of X.

4.63 Find the variance and the standard deviation. A random variable X has the following distribution:

X	−2	−1	0	1
Probability	0.1	0.2	0.4	0.3

Find the variance and the standard deviation for this random variable. Show your work.

4.64 Standard deviation of the number of aces. Refer to Exercise 4.62. Find the standard deviation of the number of aces.

4.65 Find some variances and standard deviations. Suppose that X is a random variable with mean 20 and standard deviation 3. Also suppose that Y is a random variable with mean 60 and standard deviation 2. Assume that the correlation between X and Y is zero. Find the variance and the standard deviation of the random variable Z for each of the following cases. Be sure to show your work.

(a) $Z = 33 - 8X$.

(b) $Z = 11X - 6$.

(c) $Z = X + Y$.

(d) $Z = X - Y$.

(e) $Z = -2X + 2Y$.

4.66 Standard deviation for fruits and vegetables. Refer to Exercise 4.58. Find the variance and the standard deviation for the distribution of the number of servings of fruits and vegetables.

4.67 What happens if the correlation is not zero? Suppose that X is a random variable with mean 20 and standard deviation 3. Also suppose that Y is a random variable with mean 60 and standard deviation 2. Assume that the correlation between X and Y is 0.4. Find the variance and the standard deviation of the random

variable Z for each of the following cases. Be sure to show your work.

(a) $Z = 33 - 8X$.

(b) $Z = 11X - 6$.

(c) $Z = X + Y$.

(d) $Z = X - Y$.

(e) $Z = -2X + 2Y$.

4.68 Suppose that the correlation is zero. Refer to Example 4.51 (page 248).

(a) Recompute the standard deviation for the total of the natural-gas bill and the electricity bill, assuming that the correlation is zero.

(b) Is this standard deviation larger or smaller than the standard deviation computed in Example 4.51? Explain why.

4.69 Find the mean of the sum. Figure 4.12 (page 235) displays the density curve of the sum $Y = X_1 + X_2$ of two independent random numbers, each uniformly distributed between 0 and 1.

(a) The mean of a continuous random variable is the balance point of its density curve. Use this fact to find the mean of Y from Figure 4.12.

(b) Use the same fact to find the means of X_1 and X_2. (They have the density curve pictured in Figure 4.9, page 229.) Verify that the mean of Y is the sum of the mean of X_1 and the mean of X_2.

4.70 Calcium supplements and calcium in the diet. Refer to Example 4.52 (page 249). Suppose that people who have high intakes of calcium in their diets are more compliant than those who have low intakes. What effect would this have on the calculation of the standard deviation for the total calcium intake? Explain your answer.

4.71 Toss a four-sided die twice. Role-playing games like Dungeons & Dragons use many different types of dice. Suppose that a four-sided die has faces marked 1, 2, 3, and 4. The intelligence of a character is determined by rolling this die twice and adding 1 to the sum of the spots. The faces are equally likely, and the two rolls are independent. What is the average (mean) intelligence for such characters? How spread out are their intelligences, as measured by the standard deviation of the distribution?

4.72 Means and variances of sums. The rules for means and variances allow you to find the mean and variance of a sum of random variables without first finding the distribution of the sum, which is usually much harder to do.

(a) A single toss of a balanced coin has either 0 or 1 head, each with probability 1/2. What are the mean and standard deviation of the number of heads?

(b) Toss a coin four times. Use the rules for means and variances to find the mean and standard deviation of the total number of heads.

(c) Example 4.27 (page 227) finds the distribution of the number of heads in four tosses. Find the mean and standard deviation from this distribution. Your results in parts (b) and (c) should agree.

4.73 What happens when the correlation is 1? We know that variances add if the random variables involved are uncorrelated ($\rho = 0$) but not otherwise. The opposite extreme is perfect positive correlation ($\rho = 1$). Show by using the general addition rule for variances that, in this case, the standard deviations add. That is, $\sigma_{X+Y} = \sigma_X + \sigma_Y$ if $\rho_{XY} = 1$.

4.74 Will you assume independence? In which of the following games of chance would you be willing to assume independence of X and Y in making a probability model? Explain your answer in each case.

(a) In blackjack, you are dealt two cards and examine the total points X on the cards (face cards count 10 points). You can choose to be dealt another card and compete based on the total points Y on all three cards.

(b) In craps, the betting is based on successive rolls of two dice. X is the sum of the faces on the first roll, and Y the sum of the faces on the next roll.

4.75 Transform the distribution of heights from centimeters to inches. A report of the National Center for Health Statistics says that the heights of 20-year-old men have mean 176.8 centimeters (cm) and standard deviation 7.2 cm. There are 2.54 centimeters in an inch. What are the mean and standard deviation in inches?

4.76 Fire insurance. An insurance company looks at the records for millions of homeowners and sees that the mean loss from fire in a year is $\mu = \$300$ per person. (Most of us have no loss, but a few lose their homes. The \$300 is the average loss.) The company plans to sell fire insurance for \$300 plus enough to cover its costs and profit. Explain clearly why it would be a bad business decision to sell only 5 policies. Then explain why selling thousands of such policies is a safe business.

4.77 Mean and standard deviation for 5 policies and for 20 policies. In fact, the insurance company in the previous exercise sees that in the entire population of homeowners, the mean loss from fire is $\mu = \$300$, and the standard deviation of the loss is $\sigma = \$400$. What are the mean and standard deviation of the average loss for 5 policies? (Losses on separate policies are assumed to be independent.) What are the mean and standard deviation of the average loss for 20 policies?

4.90 Exercise and sleep. Suppose that 46% of adults get enough sleep, 40% get enough exercise, and 27% do both. Find the probabilities of the following events:

(a) Enough sleep and not enough exercise.

(b) Not enough sleep and enough exercise.

(c) Not enough sleep and not enough exercise.

(d) For each of parts (a), (b), and (c), state the rule that you used to find your answer.

4.91 Venn diagram for exercise and sleep. Refer to the previous exercise. Draw a Venn diagram showing the probabilities for exercise and sleep.

4.92 Find a probability for lying to a teacher. Suppose that 46% of high school students would admit to lying at least once to a teacher during the past year and that 28% of students are male and would admit to lying at least once to a teacher during the past year.[18] Assume that 44% of the students are male. What is the probability that a randomly selected student is either male or would admit to lying to a teacher during the past year? Be sure to show your work and indicate all the rules that you use to find your answer.

4.93 Find another probability for lying to a teacher. Refer to the previous exercise. Suppose that you select a student from the subpopulation of those who would admit to lying to a teacher during the past year. What is the probability that the student is female? Be sure to show your work and indicate all the rules that you use to find your answer.

4.94 Attendance at two-year and four-year colleges. In a large national population of college students, 59% attend four-year institutions, and the rest attend two-year institutions. Males make up 44% of the students in the four-year institutions and 40% of the students in the two-year institutions.

(a) Find the four probabilities for each combination of sex and type of institution in the following table. Be sure that your probabilities sum to 1.

	Male	Female
Four-year institution		
Two-year institution		

4.97 Education and income. Call a household prosperous if its income exceeds \$100,000. Call the household educated if the householder completed college. Select an American household at random and let A be the event that the selected household is prosperous and B the event that it is educated. According to the Current Population Survey, $P(A) = 0.138$, $P(B) = 0.261$, and the probability that a household is both prosperous and educated is $P(A \text{ and } B) = 0.082$. What is the probability $P(A \text{ or } B)$ that the household selected is either prosperous or educated?

4.98 Find a conditional probability. In the setting of the previous exercise, what is the conditional probability that a household is prosperous, given that it is educated? Explain why your result shows that events A and B are not independent.

5.3 Describe the population and the sample. For each of the following situations, describe the population and the sample.

(a) A survey of 18,875 people aged 18 to 25 reported that 55.1% drank alcohol in the past month.

(b) In a study of work stress, 250 restaurant workers were asked about the impact of work stress on their personal lives.

(c) In a study of Monarch butterflies, 55 milkweed plants in a Yosemite Valley were randomly sampled. The average number of Monarch eggs per plant was 0.73.

5.4 Is it unbiased? A statistic has a sampling distribution that is somewhat skewed. The mean is 20.0, the median is 19.3, and the quartiles are 15.3 and 23.9.

(a) If the true parameter value is 19.3, is the estimator unbiased?

(b) If the true parameter value is 20.0, is the estimator unbiased?

(c) If the true parameter value is 19.6, is the estimator unbiased?

(d) Write a short summary of your results in parts (a), (b), and (c) and include a discussion of bias and unbiased estimators.

5.5 Constructing a sampling distribution. Refer to Example 5.1 (page 272). Suppose *Student Monitor* also reported that the median number of weekly hours per course spent outside of class was 2.5 hours.

(a) Explain why we'd expect the population median to be less than the population mean in this setting by drawing the distribution of weekly hours per course spent outside of class for all undergraduates. This is called the *population distribution*.

(b) Using Figure 5.2 (page 274) as a guide and your distribution from part (a), describe how to approximate the sampling distribution of the sample median in this setting.

5.6 Bias and variability. FIGURE 5.5 shows histograms of four sampling distributions of statistics intended to estimate the same parameter. Label each distribution relative to the others as high or low bias and as high or low variability.

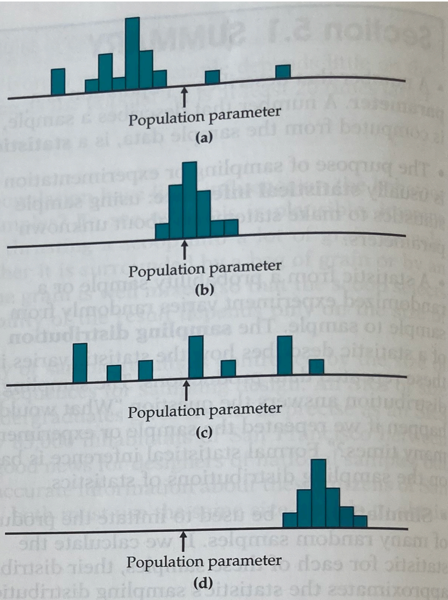

FIGURE 5.5 Determine which of these sampling distributions displays high or low bias and high or low variability, Exercise 5.6.

proportions (count of heads divided by 25). You are constructing the sampling distribution of this statistic.

(b) Another population contains only 15% who plan to vote in the next election. Take 50 samples of size 25 from this population, record the number in each sample who approve, and make a histogram of the 50 sample proportions.

5.8 Comparing sampling distributions. Refer to the previous exercise.

(a) How do the centers of your two histograms reflect the differing truths about the two populations?

(b) Describe any differences in the shapes of the two histograms. Is one more skewed than the other?