Section 5.2 SUMMARY

- The **population distribution** of a variable X is the distribution of its values for all members of the population. This distribution and the sample size n affect the distribution of $\bar{x}$.
- The sample mean $\bar{x}$ of an SRS of size n drawn from a large population with mean μ and standard deviation σ has a sampling distribution with mean and standard deviation

$$\mu_{\bar{x}} = \mu$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

- The sample mean $\bar{x}$ is an **unbiased estimator** of the population mean μ and is less variable than a single observation.
- The standard deviation of $\bar{x}$ decreases in proportion to the square root of the sample size n. This means that to reduce the standard deviation by a factor of C, we need to increase the sample size by a factor of C^2.
- The **central limit theorem** states that, for large n, the sampling distribution of $\bar{x}$ is approximately $(\mu, \sigma/\sqrt{n})$ for any population with mean μ and finite standard deviation σ. This allows us to approximate probability calculations of $\bar{x}$ using the Normal distribution.
- Linear combinations of independent Normal random variables have Normal distributions. In particular, if the population has a Normal distribution, so does $\bar{x}$.

Now that you have completed this section, you will be able to:

- Explain the difference between the sampling distribution of $\bar{x}$ and the population distribution. *Review Example 5.5 (page 283) and try Exercise 5.15.*
- Determine the mean and standard deviation of $\bar{x}$ for an SRS of size n from a population with mean μ and standard deviation σ. *Review Example 5.6 (page 286) and try Exercises 5.19 and 5.23.*
- Describe how many times larger n has to be for an SRS to reduce the standard deviation of $\bar{x}$ by a certain factor. *Review Example 5.8 (page 287) and try Exercise 5.17.*
- Utilize the central limit theorem to approximate the sampling distribution of $\bar{x}$ and perform probability calculations based on this approximation. *Review Example 5.11 (page 290) and try Exercise 5.25.*

Section 5.2 EXERCISES

5.13 What's wrong? For each of the following statements, explain what is wrong and why.

(a) If the population standard deviation is 20, then the standard deviation of $\bar{x}$ for an SRS of 10 observations is $20/10 = 2$.

(b) When taking SRSs from a population, larger sample sizes will result in larger standard deviations of $\bar{x}$.

(c) For an SRS from a population, both the mean and the standard deviation of $\bar{x}$ depend on the sample size n.

(d) The larger the population, the bigger the sample size n needs to be for a desired standard deviation of $\bar{x}$.

5.14 What's wrong? For each of the following statements, explain what is wrong and why.

(a) The central limit theorem states that for large n, the population mean μ is approximately Normal.

(b) For large n, the distribution of observed values will be approximately Normal.

(c) For sufficiently large n, the 68–95–99.7 rule says that $\bar{x}$ should be within 2σ of μ about 95% of the time.

(d) Refer to Figure 5.13. For $\bar{x}$ to be approximately Normal, we will need to draw a larger sample size n from the distribution in panel (c) than in panel (a).

5.15 Generating a sampling distribution. Let's illustrate the idea of a sampling distribution in the case of a very small sample from a very small population. The population is the 10 scholarship players currently on your women's basketball team. For convenience, the 10 players have been labeled with the integers 0 to 9. For each player, the total amount of time spent (in minutes) on Twitter during the past week is recorded in the following table.

Player	0	1	2	3	4	5	6	7	8	9
Time (min)	118	24	89	85	74	135	116	107	60	99

The parameter of interest is the average amount of time on Twitter. The sample is an SRS of size $n = 3$ drawn from this population of players. Because the players are labeled 0 to 9, a single random digit from Table B chooses one player for the sample.

(a) Find the mean for the 10 players in the population. This is the population mean μ.

(b) Use Table B to draw an SRS of size 3 from this population. (*Note:* You may sample the same player's time more than once.) Write down the three times in your sample and calculate the sample mean $\bar{x}$. This statistic is an estimate of μ.

(c) Repeat this process nine more times, using different parts of Table B. Make a histogram of the 10 values of $\bar{x}$. You are approximating the sampling distribution of $\bar{x}$.

(d) Is the center of your histogram close to μ? Explain why you'd expect it to get closer to μ the more times you repeated this sampling process.

5.16 Sleep duration of college students. In Example 5.4, the daily sleep duration among college students was approximately Normally distributed with mean $\mu = 7.13$ hours and standard deviation $\sigma = 1.67$ hours. You plan to take an SRS of size $n = 60$ and compute the average total sleep time.

(a) What is the standard deviation for the average time?

(b) Use the 95 part of the 68–95–99.7 rule to describe the variability of this sample mean.

(c) What is the probability that your average will be below 6.9 hours?

5.17 Determining sample size. Refer to the previous exercise. You want to use a sample size such that about 95% of the averages fall within ±10 minutes (0.17 hour) of the true mean $\mu = 7.13$.

(a) Based on your answer to part (b) in Exercise 5.16, should the sample size be larger or smaller than 60? Explain.

(b) What standard deviation of $\bar{x}$ do you need such that approximately 95% of all samples will have a mean within 10 minutes of μ?

(c) Using the standard deviation you calculated in part (b), determine the number of students you need to sample.

5.18 Length of a movie on Netflix. Flixable reports that Netflix's U.S. catalog contains almost 4000 movies.[9] You are interested in determining the average length of these movies. Previous studies have suggested the standard deviation for this population is 34 minutes.

(a) What is the standard deviation of the average length if you take an SRS of 25 movies from this population?

(b) How many movies would you need to sample if you wanted the standard deviation of $\bar{x}$ to be no larger than 5 minutes?

5.24 Grades in a math course. Indiana University posts the grade distributions for its courses online.[11] In one spring semester, students in Math 118 received 16.1% A's, 34.3% B's, 29.2% C's, 9.6% D's, and 9.8% F's.

(a) Using the common scale A = 4, B = 3, C = 2, D = 1, F = 0, take X to be the grade of a randomly chosen Math 118 student. Use the definitions of the mean (page 237) and standard deviation (page 245) for discrete random variables to find the mean μ and the standard deviation σ of grades in this section.

(b) Math 118 is a large enough course that we can take the grades of an SRS of 25 students and not worry about the finite population correction factor. If $\bar{x}$ is the average of these 25 grades, what are the mean and standard deviation of $\bar{x}$?

(c) What is the probability that a randomly chosen Math 118 student gets a B or better, $P(X \geq 3)$?

(d) What is the approximate probability that the grade point average for 25 randomly chosen Math 118 students is B or better, $P(\bar{x} \geq 3)$?

(e) Explain why the probabilities in parts (c) and (d) are so different.

5.30 Should you use the binomial distribution? In each of the following situations, is it reasonable to use a binomial distribution for the random variable X? Give reasons for your answer in each case.

(a) In a random sample of students in a fitness study, X is the mean daily exercise time of the sample.

(b) A manufacturer of running shoes picks a random sample of 20 shoes from the production of shoes each day for a detailed inspection. X is the number of pairs of shoes with a defect.

(c) A nutrition study chooses an SRS of college students. They are asked whether or not they usually eat at least five servings of fruits or vegetables per day. X is the number who say that they do.

(d) X is the number of days during the school year when you skip a class.

5.43 Metal fatigue. Metal fatigue refers to the gradual weakening and eventual failure of metal that undergoes cyclic loads. The wings of an aircraft, for example, are subject to cyclic loads when in the air, and cracks can form. It is thought that these cracks start at large particles found in the metal. Suppose that the number of particles large enough to initiate a crack follows a Poisson distribution with mean $\mu = 0.5$ per square centimeter (cm^2).

(a) What is the mean of the Poisson distribution if we consider a 100 cm^2 area?

(b) Using the Normal approximation, what is the probability that this section has more than 60 of these large particles?

(c) Summarize the relationship between the sample size and the standard deviation that your graph shows.

5.47 Monitoring the emerald ash borer. The emerald ash borer is a beetle that poses a serious threat to ash trees. Purple traps are often used to detect or monitor populations of this pest. In the counties of your state where the beetle is present, thousands of traps are used to monitor the population. These traps are checked periodically. The distribution of beetle counts per trap is discrete and strongly skewed. A majority of traps have no beetles, and only a few will have more than two beetles. For this exercise, assume that the mean number of beetles trapped is 0.43, with a standard deviation of 0.95.

(a) Suppose that your state does not have the resources to check all the traps, so it plans to check only an SRS of $n = 150$ traps. What are the mean and standard deviation of the average number of beetles $\bar{x}$ in 150 traps?

(b) Use the central limit theorem to find the probability that the average number of beetles in 150 traps is greater than 0.55.

(c) Do you think it is appropriate in this situation to use the central limit theorem? Explain your answer.

5.48 Attitudes toward ...

5.63 A test for ESP. In a test for ESP (extrasensory perception), the experimenter looks at cards that are hidden from the subject. Each card contains either a star, a circle, a wave, or a square. As the experimenter looks at each of 20 cards in turn, the subject names the shape on the card.

(a) If a subject simply guesses the shape on each card, what is the probability of a successful guess on a single card? Because the cards are independent, the count of successes in 20 cards has a binomial distribution.

(b) What is the probability that a subject correctly guesses at least 10 of the 20 shapes?

(c) In many repetitions of this experiment with a subject who is guessing, how many cards will the subject guess correctly, on the average? What is the standard deviation of the number of correct guesses?

(d) A standard ESP deck actually contains 25 cards. There are 5 different shapes, each of which appears on 5 cards. The subject knows that the deck has this makeup. Is a binomial model still appropriate for the count of correct guesses in one pass through this deck? If so, what are n and p? If not, why not?

PUTTING IT ALL TOGETHER

5.72 Risks and insurance. The idea of insurance is that we all face risks that are unlikely but carry high cost. Think of a fire destroying your home. So we form a group to share the risk: we all pay a small amount, and the insurance policy pays a large amount to those few of us whose homes burn down. An insurance company looks at the records for millions of homeowners and sees that the mean loss from fire in a year is $\mu = \$600$ per house and that the standard deviation of the loss is $\sigma = \$12{,}000$. (The distribution of losses is extremely right-skewed: most people have $0 loss, but a few have large losses.) The company plans to sell fire insurance for $500 plus enough to cover its costs and profit.

(a) Explain clearly why it would be unwise to sell only 100 policies. Then explain why selling many thousands of such policies is a safe business.

(b) Suppose the company sells the policies for $700. If the company sells 50,000 policies, what is the approximate probability that the average loss in a year will be greater than $700?

5.73 Binge drinking. The Centers for Disease Control and Prevention finds that 28% of people aged 18 to 24 years binge drank. Those who binge drank averaged 9.3 drinks per episode and 4.2 episodes per month. The study took a sample of over 18,000 people aged 18 to 24 years, so the population proportion of people who binge drank is very close to $p = 0.28$.[28] The administration of your college surveys an SRS of 200 students and finds that 56 binge drink.

(a) What is the sample proportion of students at your college who binge drink?

(b) If, in fact, the proportion of all students on your campus who binge drink is the same as the national 28%, what is the probability that the proportion in an SRS of 200 students is as large as or larger than the result of the administration's sample?

(c) A writer for the student paper says that the percent of students who binge drink is higher on your campus than nationally. Write a short letter to the editor explaining why the survey does not support this conclusion.

5.74 The ideal number of children. "What do you think is the ideal number of children for a family to have?" A Gallup Poll asked this question of 1020 randomly chosen adults. Roughly 41% thought that a total of three or more children was ideal.[29] Suppose that $p = 0.41$ is exactly true for the population of all adults. Gallup announced a margin of error of ±4 percentage points for this poll.

(a) What is the probability that the sample proportion $\hat{p}$ for an SRS of size $n = 1020$ falls between 0.37 and 0.45? You see that it is likely, but not certain, that polls like this give results that are correct within their margin of error.

(b) What is the probability that a sample proportion $\hat{p}$ falls between 0.37 and 0.45 (that is, within ±4 percentage points of the true p) if the sample is an SRS of size $n = 300$? Of size $n = 5000$?

(c) Combine these results to make a general statement about the effect of larger samples in a sample survey.

5.75 Is the ESP result better than guessing? When the ESP study of Exercise 5.63 discovers a subject whose performance appears to be better than guessing, the study continues at greater length. The experimenter looks at many cards bearing one of five shapes (star, square, circle, wave, and cross) in an order determined by random numbers. The subject cannot see the experimenter as the experimenter looks at each card in turn, in order to avoid any possible nonverbal clues. The answers of a subject who does not have ESP should be independent observations, each with probability 1/5 of success. We record 900 attempts.

(a) What are the mean and the standard deviation of the count of successes?

(b) What are the mean and the standard deviation of the proportion of successes among the 900 attempts?

(c) What is the probability that a subject without ESP will be successful in at least 24% of 900 attempts?

(d) The researcher considers evidence of ESP to be a proportion of successes so large that there is only probability 0.01 that a subject could do this well or better by guessing. What proportion of successes must a subject have to meet this standard? (Example 1.45, on page 62, shows how to do an inverse calculation for the Normal distribution that is similar to the type required here.)

5.76 How large a sample is needed? The changing probabilities you found in Exercise 5.74 are due to the fact that the standard deviation of the sample proportion $\hat{p}$ gets smaller as the sample size n increases. If the population proportion is $p = 0.41$, how large a sample is needed to reduce the standard deviation of $\hat{p}$ to $\sigma_{\hat{p}} = 0.005$? (The 68–95–99.7 rule then says that about 95% of all samples will have $\hat{p}$ within 0.01 of the true p.)