

6.5 Confidence interval mistakes and misunderstandings.

Suppose that 500 randomly selected alumni of the University of Okoboji were asked to rate the university's academic advising services on a scale of 1–10. The sample mean \bar{x} was found to be 8.6. Assume that the population standard deviation is known to be $\sigma = 2.2$.

- Ima Bitlost computes the 95% confidence interval for the average satisfaction score as $8.6 \pm 1.96(2.2)$. What is her mistake?
- After correcting her mistake in part (a), Ima states, "I am 95% confident that the sample mean falls between 8.4 and 8.8." What is wrong with this statement?
- Ima quickly realizes her mistake in part (b) and instead states, "The probability that the true mean is between 8.4 and 8.8 is 0.95." What misinterpretation is she making now?
- Finally, in her defense for using the Normal distribution to determine the confidence interval, Ima says, "Because the sample size is quite large, the population of alumni ratings will be approximately Normal." Explain to Ima her misunderstanding and correct this statement.

6.9 Mean TRAP in young women. For many important processes that occur in the body, direct measurement of characteristics of the process is not possible. In many cases, however, we can measure a *biomarker*, a biochemical substance that is relatively easy to measure and is associated with the process of interest. Bone turnover is the net effect of two processes: the breaking down of old bone, called resorption, and the building of new bone, called formation. One biochemical measure of bone resorption is tartrate-resistant acid phosphatase (TRAP), which can be measured in blood. In a study of bone turnover in young women, serum TRAP was measured in 31 subjects.⁹ The mean was 13.2 units per liter (U/l). Assume that the standard deviation is known to be 6.5 U/l. Give the margin of error and find a 95% confidence interval for the mean TRAP amount in young women represented by this sample.

6.10 Mean OC in young women. Refer to the previous exercise. A biomarker for bone formation measured in the same study was osteocalcin (OC), measured in the blood. For the 31 subjects in the study, the mean was 33.4 nanograms per milliliter (ng/ml). Assume that the standard deviation is known to be 19.6 ng/ml. Report the 95% confidence interval.

6.21 Required sample size for specified margin of error.

A new bone study is being planned that will measure the biomarker TRAP described in Exercise 6.9. Using the value of σ given there, 6.5 U/l, find the sample size required to provide an estimate of the mean TRAP with a margin of error of 1.5 U/l for 95% confidence.

6.24 Accuracy of a laboratory scale. To assess the accuracy of a laboratory scale, a standard weight known to weigh 10 grams is weighed repeatedly. The scale readings are Normally distributed with unknown mean. (This mean is 10 grams if the scale has no bias.) The standard deviation of the scale readings is known to be 0.0002 gram.

- The weight is measured six times. The mean result is 10.0023 grams. Give a 99% confidence interval for the mean of repeated measurements of the weight.
- Based on the interval in part (a), do you think the scale is accurate? Explain your answer.
- How many measurements must be averaged to get a margin of error of ± 0.0001 with 99% confidence?