EXAMPLE 10.7

DATA PABMI

Confidence intervals for the mean response. FIGURE 10.7 shows the upper and lower confidence limits on a graph with the data and the least-squares line. The 95% confidence limits appear as dashed curves. For any x^*, the confidence interval for the mean response extends from the lower dashed curve to the upper dashed curve. The intervals are narrowest for values of x^* near the mean of the observed x's and widen as x^* moves away from $\bar{x}$.

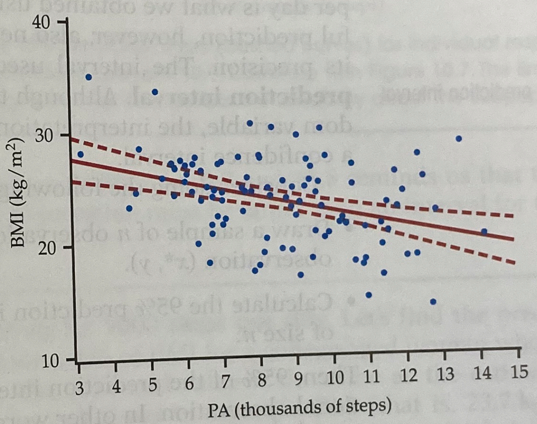

FIGURE 10.7 The 95% confidence limits (dashed curves) for the mean response for the physical activity study, Example 10.7.

Some software will do these calculations directly if you input a value for the explanatory variable. Other software will calculate the intervals for each value of x in the data set. Creating a new data set with an additional observation with x equal to the value of interest and y missing will often work.

EXAMPLE 10.8

Confidence interval for an average of 9000 steps per day. Let's find the confidence interval for the average BMI at $x = 9.0$. Our predicted BMI is

$$\widehat{\text{BMI}} = 29.578 - (0.655 \times \text{PA})$$
$$= 29.578 - (0.655 \times 9.0)$$
$$= 23.7$$

Software tells us that the 95% confidence interval for the mean response is 23.0 to 24.4 kg/m^2.

...an who averaged 9000 steps per day, we would ...d 24.4 kg/m^2. Note that many

EXAMPLE 10.10

Prediction interval for 9000 steps per day. Let's find the prediction interval for a future observation of BMI for a college-aged woman who averages 9000 steps per day. The predicted value is the same as the estimate of the average BMI that we calculated in Example 10.8 (that is, 23.7 kg/m^2). Software tells us that the 95% prediction interval is 16.4 to 31.0 kg/m^2. This interval is extremely wide, covering BMI values that are classified as underweight as well as obese. Because of the large amount of scatter about the regression line, prediction intervals here are relatively useless.

Although a larger sample would better estimate the population regression line, it would not reduce the degree of scatter about the line. This means that prediction intervals for BMI, given activity level, will *always* be wide. This example clearly demonstrates that a very small *P*-value for the significance test for a zero slope does not necessarily imply that we have found a strong predictive relationship.

CHECK-IN

10.6 Margin of error for the predicted mean. Refer to Figure 10.7 (page 529) and Example 10.8 (page 529). What is the 95% margin of error of $\hat{\mu}_y$ when $x = 9.0$? Would you expect the margin of error to be larger, smaller, or the same for $x = 11.0$? Explain your answer.

10.7 Margin of error for a predicted response. Refer to Example 10.10. What is the 95% margin of error of $\hat{y}$ when $x = 9.0$? If you increased the sample size from $n = 100$ to $n = 400$, would you expect the 95% margin of error for the predicted response to be roughly twice a large, half as large, or the same for $x = 9.0$? Explain your answer.

...forming variables

...ample 10.1 with a scatterplot to check whether