Tilleggsoppgaver for STK1100 Vår 2016

Matematisk institutt

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Tilleggsoppgave 24
Anta X og Y er to diskrete stokastiske variable med simultan punktsannsynlighet gitt ved

$$p(x,y) = \begin{cases} \frac{x+y}{32} & x = 1,2, y = 1,2,3,4 \\ 0 & \text{ellers} \end{cases}$$

(a) Da er $P(X = 2)$ lik
A: $\frac{7}{16}$ B: $\frac{1}{2}$ C: $\frac{9}{16}$ D: $\frac{2+y}{32}$ E: $\frac{1+y}{32}$
Løsning:

$$\begin{aligned} P(X = 2) =& P(X = 2, Y = 1) + P(X = 2, Y = 2) + P(X = 2, Y = 3) + P(X = 2, Y = 4) \\ =& \frac{3}{32} + \frac{4}{32} + \frac{5}{32} + \frac{6}{32} \\ =& \frac{18}{32} = \frac{9}{16} \end{aligned}$$

(b) $P(Y = 2X)$ er
A: $\frac{1}{4}$ B: $\frac{5}{32}$ C: $\frac{1}{2}$ D: $\frac{3}{32}$ E: $\frac{9}{32}$
Løsning:

$$\begin{aligned} P(Y = 2X) =& P(X = 1, Y = 2) + P(X = 2, Y = 4) \\ =& \frac{3}{32} + \frac{6}{32} \\ =& \frac{9}{32} \end{aligned}$$

(c) Sannsynligheten for at $X \leq Y$ er
A: $\frac{1}{2}$ B: $\frac{3}{32}$ C: $\frac{29}{32}$ D: $\frac{3}{16}$ E: $\frac{3}{8}$
Løsning:

$$\begin{aligned} P(X \leq Y) =& 1 - P(X > Y) \\ =& 1 - P(X = 2, Y = 1) \\ =& 1 - \frac{3}{32} = \frac{29}{32} \end{aligned}$$

Tilleggsoppgave 25
Anta X og Y er to kontinuerlig stokastiske variable med simultan sannsynlighetstetthet gitt ved

$$f(x,y) = \begin{cases} x+y & 0 \le x \le 1, 0 \le y \le 1 \\ 0 & \text{ellers} \end{cases}$$

(a) Den marginale fordeling til X er
A: 1   B: $x + \frac{1}{2}$ C: $\frac{1}{2}$ D: $x + \frac{1}{2}x^2$ E: $y + \frac{1}{2}$

Løsning:

$$\begin{aligned} f_X(x) &= \int_0^1 f(x,y)dy \\ &= \int_0^1 (x+y)dy \\ &= [xy + \tfrac{1}{2}y^2]_0^1 \\ &= x + \tfrac{1}{2} \end{aligned}$$

(b) $E(Y)$ er
A: $\frac{1}{3}$ B: $\frac{x}{2}$ C: $\frac{7}{12}$ D: $\frac{3}{8}$ E: y

Løsning: Pga symmetri blir $f_Y(y) = y + \frac{1}{2}$. Dermed blir

$$\begin{aligned} E(Y) &== \int_0^1 y f_Y(y)dy \\ &= \int_0^1 y[y + \tfrac{1}{2}]dy \\ &= [\tfrac{1}{3}y^3 + \tfrac{1}{4}y^2]_0^1 \\ &= [\tfrac{1}{3} + \tfrac{1}{4}] \\ &= \tfrac{7}{12} \end{aligned}$$

(c) Sannsynligheten for at $X + Y > 1$ er
A: $-\frac{1}{2}$ B: $\frac{5}{8}$ C: $\frac{2}{3}$ D: $\frac{3}{4}$ E: $\frac{3}{8}$

Løsning: Pga symmetri blir $f_Y(y) = y + \frac{1}{2}$. Dermed blir

$$\begin{aligned}
P(X+Y>1) == & \int_0^1 \int_{1-x}^1 f(x,y)dydx \\
= & \int_0^1 \int_{1-x}^1 (x+y)dydx \\
= & \int_0^1 [xy + \tfrac{1}{2}y^2]_{1-x}^1 dx \\
= & \int_0^1 [x + \tfrac{1}{2} - x(1-x) - \tfrac{1}{2}(1-x)^2]dx \\
= & \int_0^1 [x + \tfrac{1}{2} - x + x^2 - \tfrac{1}{2} + x - \tfrac{1}{2}x^2]dx \\
= & \int_0^1 [\tfrac{1}{2}x^2 + x]dx \\
= & [\tfrac{1}{6}x^3 + \tfrac{1}{2}x^2]_0^1 \\
= & [\tfrac{1}{6} + \tfrac{1}{2}]_0^1 \\
= & \tfrac{4}{6} = \tfrac{2}{3}
\end{aligned}$$

(d) $E(X+Y)$ er

A: 1 B: 4 C: $\frac{7}{10}$ D: $\frac{7}{6}$ E: y

Løsning: Vi har at $E(X+Y) = E(X) + E(Y)$. Videre, pga symmetri er $E(X) = E(Y)$. Dermed er $E(X+Y) = 2 * \frac{7}{12} = \frac{7}{6}$

Tilleggsoppgave 26

Anta X og Y er to kontinuerlig stokastiske variable med simultan sannsynlighetstetthet gitt ved

$$f(x,y) = \begin{cases} kxy^2 & 0 \le x \le 2, 0 \le y \le 2 \\ 0 & \text{ellers} \end{cases}$$

(a) Da er k lik

A: $\frac{5}{16}$ B: $\frac{3}{16}$ C: $\frac{1}{16}$ D: $\frac{1}{4}$ E: $\frac{7}{16}$

Løsning: Vi har at

$$\begin{aligned}
\int_0^2 \int_0^2 xy^2 dydx = & \int_0^2 x[\tfrac{1}{3}y^3]_0^2 dx \\
= & \int_0^2 x\tfrac{8}{3}dx \\
= & \tfrac{8}{3}[\tfrac{1}{2}x^2]_0^2 \\
= & \tfrac{8}{3}\tfrac{4}{2} = \tfrac{16}{3}
\end{aligned}$$

Dermed må $k = \frac{3}{16}$ for at tettheten skal integrere seg opp til 1.

(b) Den marginale fordeling til X er
A: 1 B: $x+\frac{1}{2}$ C: $\frac{1}{2}x$ D: $x+\frac{1}{2}x^2$ E: $y+\frac{1}{2}$

Løsning:

$$\begin{aligned} f_X(X) = &\int_0^2 kxy^2 dy \\ =&k[x\tfrac{1}{3}y^3]_0^2 \\ =&kx\tfrac{8}{3} \\ =&\tfrac{1}{2}x \end{aligned}$$

(c) $E(Y)$ er
A: 1 B: $\frac{5}{12}$ C: $\frac{3}{2}$ D: $\frac{7}{12}$ E: y

Løsning:

$$\begin{aligned} f_Y(y) = &\int_0^2 \tfrac{3}{16}xy^2 dx \\ =&\tfrac{3}{16}y^2[\tfrac{1}{2}x^2]_0^2 \\ =&\tfrac{3}{16}\tfrac{4}{2}y^2 = \tfrac{3}{8}y^2 \end{aligned}$$

Dermed blir

$$\begin{aligned} E(Y) = &\int_0^2 y\tfrac{3}{8}y^2 dy \\ =&\int_0^2 \tfrac{3}{8}y^3 dy \\ =&\tfrac{3}{8}[\tfrac{1}{4}y^4]_0^2 \\ =&\tfrac{3}{8}\tfrac{16}{4} = \tfrac{3}{2} \end{aligned}$$