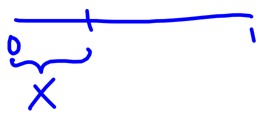
Eksempel "broken stick" på bruk av $E(h(X))$



1 enhet lang
knekker på tilfeldig sted

X : lengden fra venstre til knekkpunktet

$X \sim \text{Unif}[0,1]$ dvs. $f(x) = \begin{cases} 1 & x \in [0,1] \\ 0 & \text{ellers} \end{cases}$

Y : lengden av den lengste biten

Vil finne $E(Y)$ – nå er $Y = h(X)$

$$Y = \begin{cases} 1-X & \text{hvis } X \leq 1/2 \\ X & \text{hvis } X > 1/2 \end{cases} \leftarrow$$

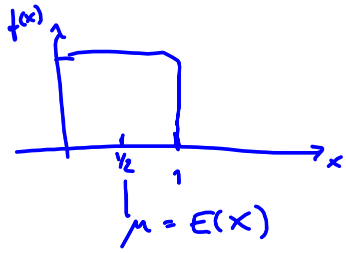
$$E(Y) = E(h(X)) = \int_{-\infty}^{\infty} h(x) f(x)\, dx$$

$$= \int_0^{1/2} (1-x) \cdot 1\, dx + \int_{1/2}^{1} x \cdot 1\, dx$$

$$= x - \frac{1}{2}x^2 \Big]_0^{1/2} + \frac{1}{2}x^2 \Big]_{1/2}^{1} = \underline{\underline{\frac{3}{4}}}$$

Varians i uniform fordeling

$X \sim \text{Unif}[0,1]$



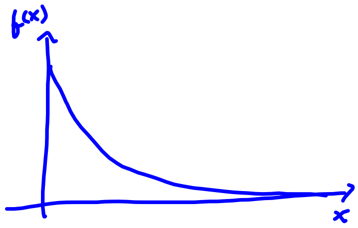
$$E(X) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_0^1 x \cdot 1\,dx$$

$$= \frac{1}{2}x^2\Big]_0^1 = 1/2$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_0^1 x^2 \cdot 1\,dx = \frac{1}{3}x^3\Big]_0^1 = 1/3$$

$$V(X) = E(X^2) - [E(X)]^2 = \frac{1}{3} - \left(\frac{1}{2}\right)^2 = \underline{\underline{\frac{1}{12}}}$$

Eksempel MGF i eksponential fordeling



$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0, \ \lambda > 0 \\ 0 & \text{ellers} \end{cases}$$

$$M_X(t) = E(e^{tX}) = \int_{-\infty}^{\infty} e^{tx} f(x)\,dx = \int_0^{\infty} e^{tx} \lambda e^{-\lambda x}\,dx$$

$$= \lambda \int_0^{\infty} e^{-(\lambda - t)x}\,dx = \lambda \left[-\frac{1}{\lambda - t} e^{-(\lambda - t)x} \right]_0^{\infty}$$

$$= \underline{\underline{\frac{\lambda}{\lambda - t}}}$$

$$M_X(t) = \frac{\lambda}{\lambda - t} = \lambda(\lambda - t)^{-1}$$

$$M_X'(t) = \frac{\lambda \cdot (-1) \cdot (-1)}{(\lambda - t)^2} = \frac{\lambda}{(\lambda - t)^2} = \lambda(\lambda - t)^{-2} \quad \leftarrow$$

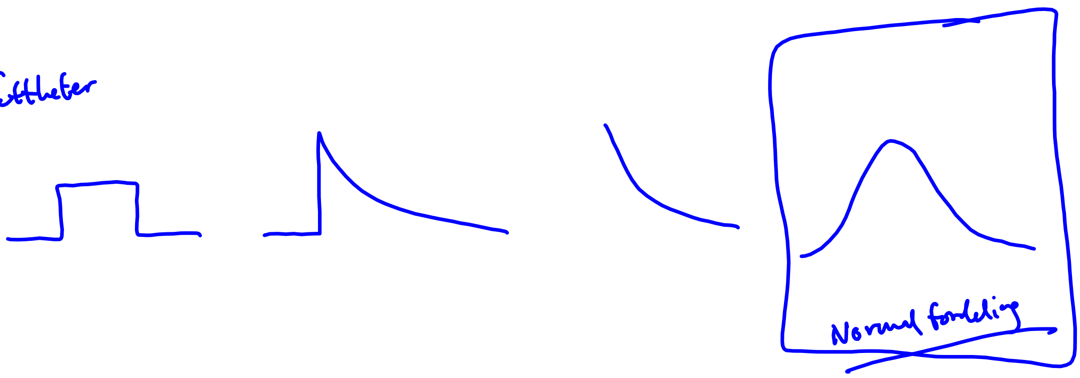
$$M_X''(t) = (-2)\lambda(\lambda - t)^{-3}(-1) = \frac{2\lambda}{(\lambda - t)^3} \quad \leftarrow$$

$$E(X) = M_X'(0) = \frac{\lambda}{(\lambda - 0)^2} = \frac{\lambda}{\lambda^2} = \underline{\underline{\frac{1}{\lambda}}}$$

$$E(X^2) = M_X''(0) = \frac{2\lambda}{(\lambda - 0)^3} = \frac{2\lambda}{\lambda^3} = \frac{2}{\lambda^2}$$

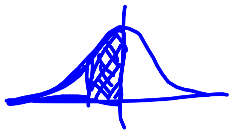
$$V(X) = E(X^2) - [E(X)]^2 = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2 = \underline{\underline{\frac{1}{\lambda^2}}}$$

Tettheter



Normalfordeling - beregning av sannsynligheter

Fødselsvekt for fullbårne jenter $= X$



$$X \sim N(3.5,\ 0.48^2)$$

Da vil vi ha $$Z = \frac{X-\mu}{\sigma} = \frac{X-3.5}{0.48} \sim N(0,1)$$

Vil finne

$$P(3.0 \le X \le 3.5) = P\left(\frac{3.0-3.5}{0.48} \le \frac{X-3.5}{0.48} \le \frac{3.5-3.5}{0.48}\right)$$

$$= P(-1.04 \le Z \le 0) = P(Z \le 0) - P(Z \le -1.04)$$

$$= \Phi(0) - \Phi(-1.04)$$

tabell

$$= 0.5 - 0.149 = \underline{0.351}$$