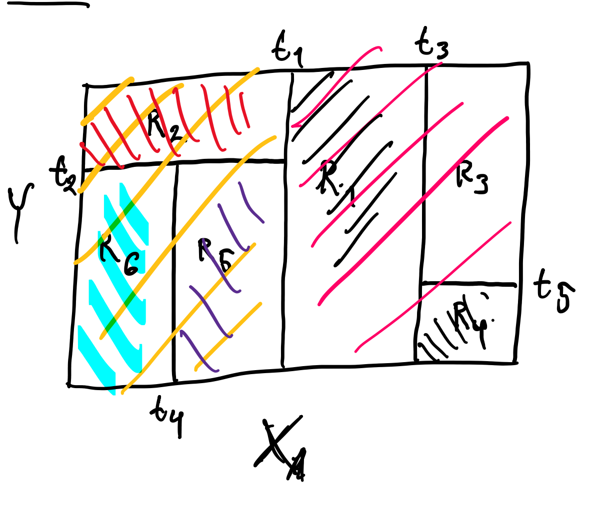
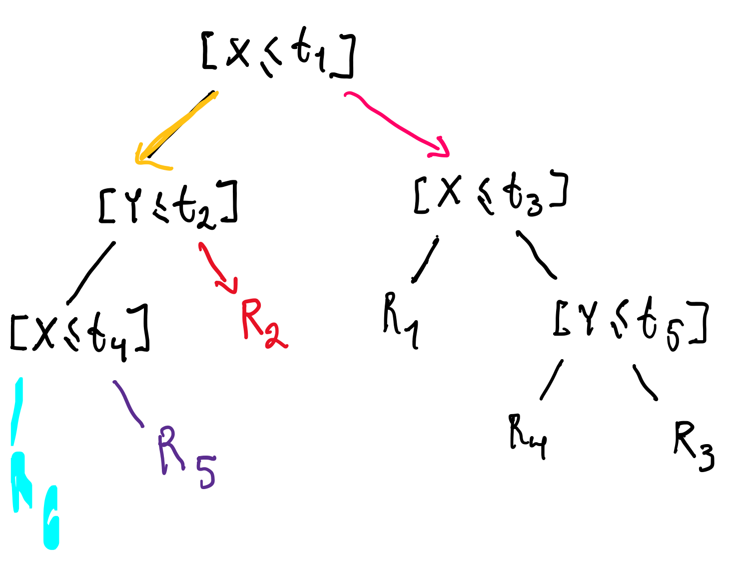
Week13-notes

Monday, 12 April 2021 12:13

# ISLR

## 8.1




## Extra Exercise 14

Bayes' classifier is a classifier that minimises the probability of missclassification (Error rate).

=> maximizing the probability of correct classification.

Bayes' theorem

$$\boxed{P(Y|X)} = \frac{P(Y,X)}{P(X)} = \frac{P(X|Y)\,P(Y)}{P(X)}$$

We are given

$P(Y) = \frac{1}{3}$, where $Y=1, Y=2$, or $Y=3$.

and $\lambda_1 = 10$, $\lambda_2 = 15$ and $\lambda_3 = 20$

$$Pr(X|Y=k) = \text{Poisson}(\lambda_k) = \frac{(5+5k)^x e^{-(5+5k)}}{x!} = \frac{5^x (1+k)^x e^{-5(1+k)}}{x!}$$

Find the marginal dist. of X.

$$P(X) = \sum_{k=1}^{3} P(X|Y=k)\underbrace{P(Y=k)}_{1/3}$$

$$= \frac{1}{3}\left[\frac{5^x(1+1)^x e^{-5(1+1)}}{x!} + \frac{5^x(1+2)^x e^{-5(1+2)}}{x!} + \frac{5^x(1+3)^x e^{-5(1+3)}}{x!}\right]$$

$$= \frac{1}{3}\left[\frac{5^x 2^x e^{-10}}{x!} + \frac{5^x 3^x e^{-15}}{x!} + \frac{5^x 4^x e^{-20}}{x!}\right]$$

$$\rightarrow = \frac{5^x e^{-10}}{3 \cdot (x!)}\left[2^x + 3^x e^{-5} + 4^x e^{-10}\right],$$

We get following cond. dist.

$$P(Y=k|X) = \frac{\frac{5^x(1+k)^x e^{-5(1+k)}}{x!}\cdot\frac{1}{3}}{\frac{5^x e^{-10}}{3\cdot(x!)}\left[2^x+3^x e^{-5}+4^x e^{-10}\right]} = \frac{(1+k)^x e^{5(1-k)}}{2^x+3^x e^{-5}+4^x e^{-10}}.$$

$$\frac{e^{-5(1+k)}}{e^{-10}} = e^{-5-5k+10} = e^{5-5k} = e^{5(1-k)}$$

We the get the following Bayes' classifier.

$$\hat{Y} \overset{def}{=} \arg\max_k Pr(Y=k|X)$$

$$= \arg\max_k \left\{\frac{(1+k)^x e^{5(1-k)}}{2^x+3^x e^{-5}+4^x e^{-10}}\right\}$$

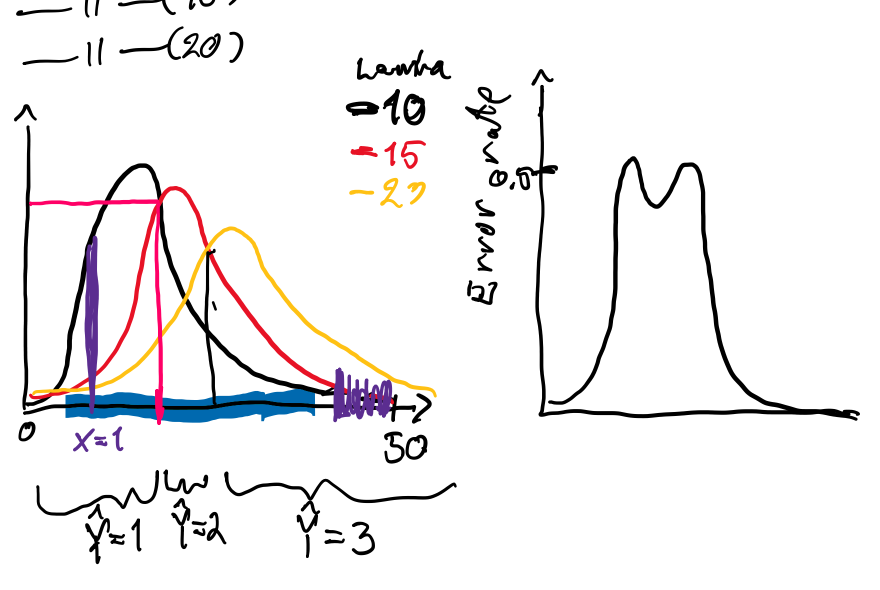
$$= \arg\max_k \left\{(1+k)^x e^{5(1-k)}\right\}$$

↑ still depend on x!

b) Error rate of Bayes classifier:

$$P(Y \ne \hat{Y}|X) = 1 - P(Y=\hat{Y}|X) = 1 - \max_k P(Y=k|X)$$

Poisson(10)
— " —(15)
— " —(20)

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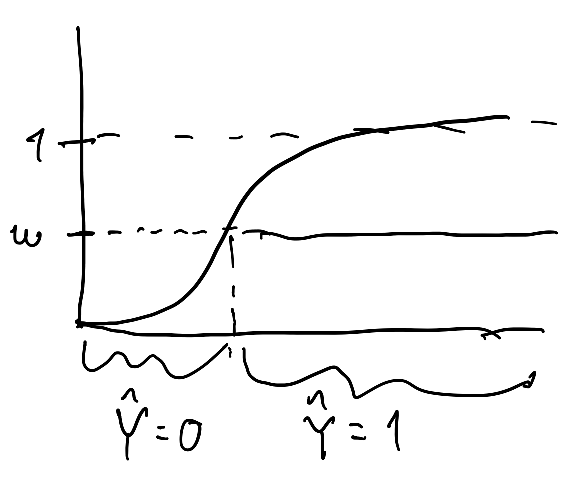
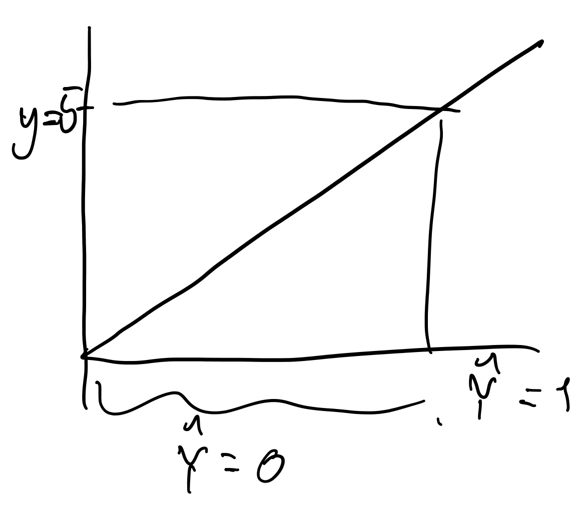
$y = \beta_0 + \beta_1 x_1 + \dots + \beta_p x_p$

$$\text{logit} = \frac{e^y}{1+e^y}$$

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~~GAM:~~

Linear regression

$E[y|x] = \hat{y} = \beta_0 + \beta_1 x_1 + \dots \beta_p x_p = \hat{\mu}$

GLM:

$g(\underbrace{E[Y|X]}_{\mu}) = \beta_0 + \beta_1 x_1 + \dots + \beta_p x_p = \eta$

↑ Link function, ↑ linear predictor

F.ex. be $\text{logit}(\eta) = \frac{e^{\eta}}{1+e^{\eta}}$ ← Binomial regre.

$\log(\eta) =$ ← Poisson regression

$g(\eta) = I(\eta) = \eta$ ← ordinary reg.

GAM:


$g(E[Y|X]) = \beta_0 + f_1(x_1) + \dots + f(x_p)$

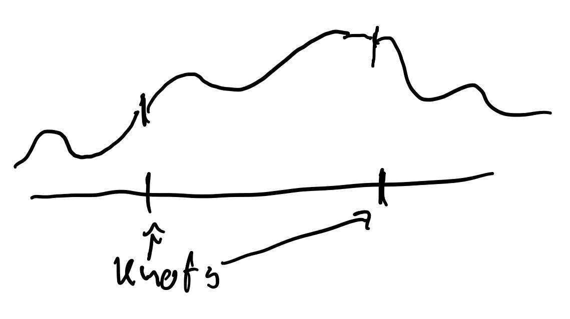
If we use $f_j(x_j) = \beta_j x_j$ we go back to glm.

In assignment we use "gam" library. (could also use "mgcv"). Both fit GAM.

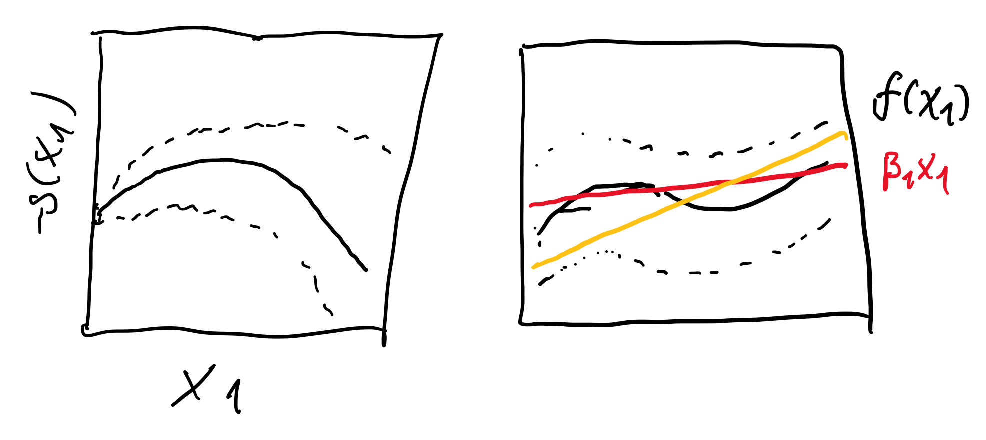
gam(y ~ s(x_1) + s(x_2) + s(x_3))

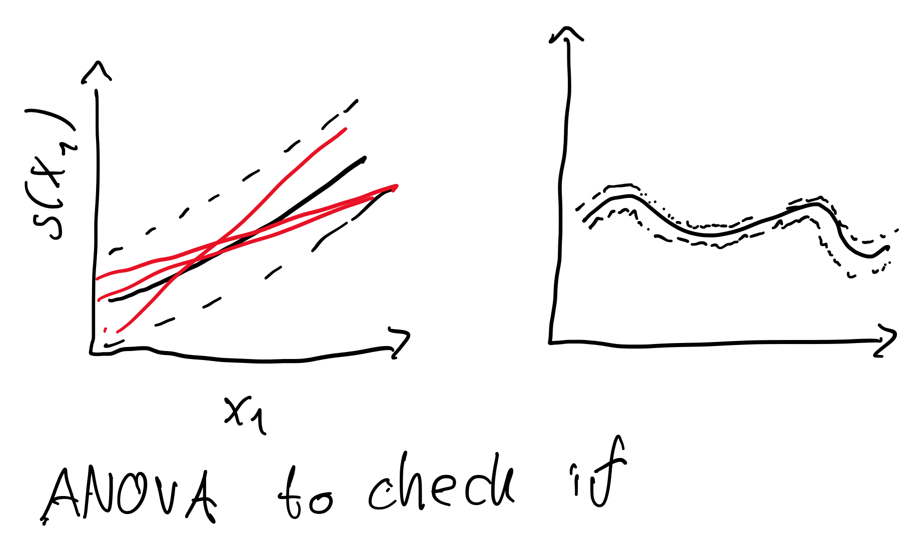
↑ smooth function.

For example be spliner

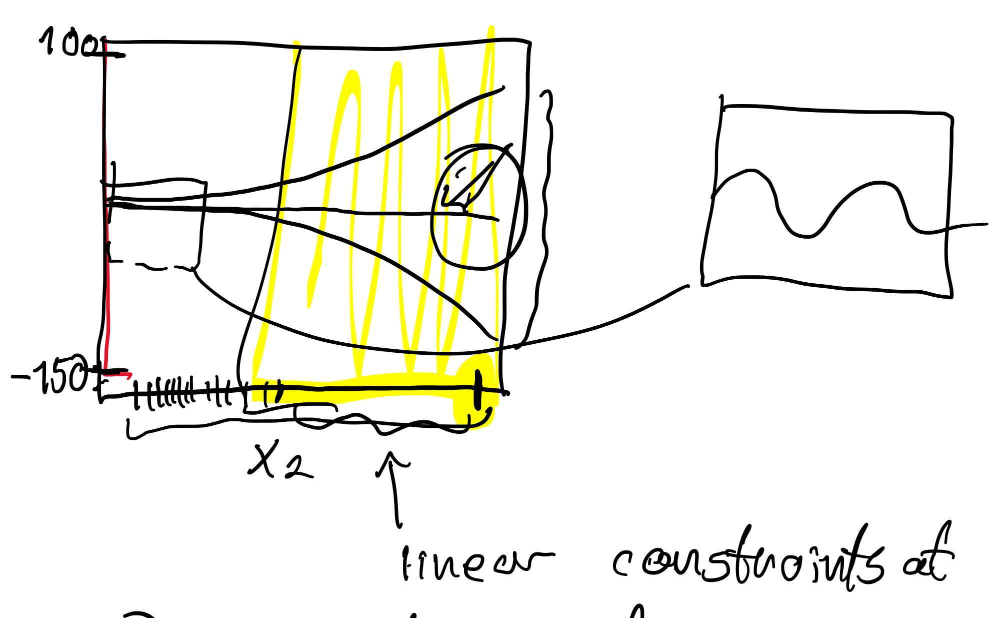



1e) plot(fit.gam) summary(fit.gamma)

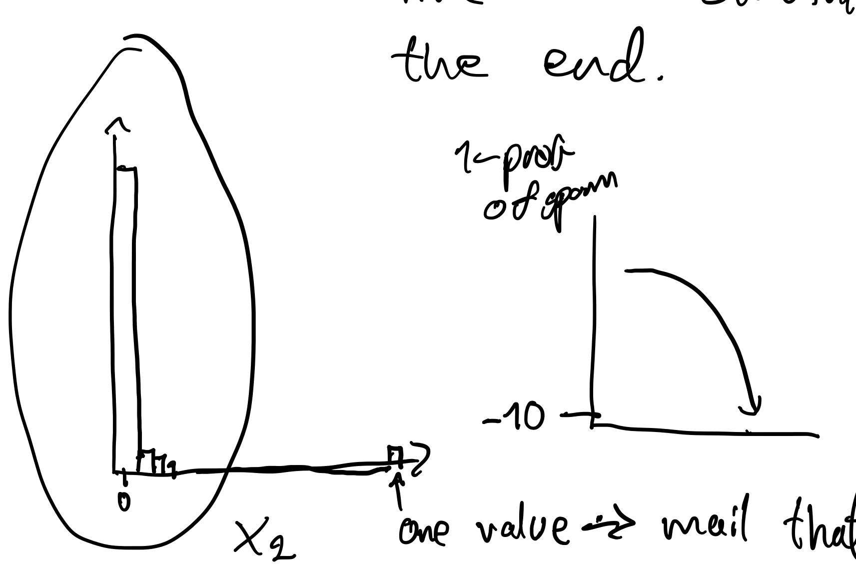



ANOVA to check if




↑ linear constraints at the end.




↑ one value → mail that is spam