Week15-notes

Monday, 26 April 2021 12:15

Extra

Ex 6.

Rewrite the loss function to

$$L(y, f(x)) = c_0 I\{y=0, f(x)=1\} + c_1 I\{y=1, f(x)=0\}$$

$$EPE(f) = E[L(Y, f(X))]$$

$$= \int_x \int_y L(y, f(x)) \underbrace{p(y,x)}_{p(y|x)p(x)} dy\, dx$$

$$= \int_x \int_y \Big[\underbrace{c_0 I\{y=0, f(x)=1\}}+\underbrace{c_1 I\{y=1, f(x)=0\}}\Big] p(y|x) dy\, \overbrace{p(x) dx}$$

$$= \int_x \int_y c_0 I\{y=0, f(x)=1\} p(y|x) dy\, p(x) dx + \int_x \int_y c_1 I\{y=1, f(x)=0\} p(y|x) dy\, p(x) dx$$

$$= \int_{x: f(x)=1} \int_y \boxed{c_0} I\{y=0\} p(y|x) dy\, p(x) dx + \int_{x: f(x)=0} \int_y \boxed{c_1} I\{y=1\} p(y|x) dy\, p(x) dx$$

$$= \int_{x: f(x)=1} \underbrace{c_0 P(Y=0|X=x)}_{Q_0} p(x) dx + \int_{x: f(x)=0} \underbrace{c_1 P(Y=1|X=x)}_{Q_1} p(x) dx$$

$$= \int_{x: f(x)=1} Q_0(x) p(x) dx + \int_{x: f(x)=0} Q_1(x) p(x) dx$$

$$= \int_X I\{f(x)=1\} Q_0(x) p(x) dx + \int_X I\{f(x)=0\} Q_1(x) p(x) dx$$

$$= \int_X \Big[\underbrace{I\{f(x)=1\} Q_0(x) + I\{f(x)=0\} Q_1(x)}\Big] p(x) dx$$

Could have used that $I\{f(x)=1\} = 1 - I\{f(x)=0\}$ Easier!

$$= \int_X \Big[\text{———} \; '' \; \text{———} + I\{f(x)=0\} Q_0(x) - I\{f(x)=0\} Q_0(x) \Big] p(x) dx$$

$$= \int_X \Big[I\{f(x)=0\}(Q_1(x) - Q_0(x)) + Q_0(x)(\underbrace{I\{f(x)=0\} + I\{f(x)=1\}}_{=1}) \Big] p(x) dx$$

$$= \int_X Q_0(x) p(x) dx + \int_X [I\{f(x)=0\}(Q_1(x) - Q_0(x))] p(x) dx$$

Constant since it does not depend on the choice of f.

$$= \text{Constant} + \int_X I\{f(x)=0\}(Q_1(x) - Q_0(x)) p(x) dx$$

We want to minimize EPE, that is the same as minimizing

$I\{f(x)=0\}(Q_1(x) - Q_0(x))$ (*)

Recall that we can only alter this expression by changing f.

So, if $Q_1(x) - Q_0(x) > 0$, we should set $f(x)=1$, then (*) will be 0.

If $Q_1(x) - Q_0(x) < 0$, we should set $f(x) = 0$, as we then get a negative result, and lower $EPE(f)$.

In other words, we set $f(x)=1$ if

$$Q_1(x) - Q_0(x) > 0$$

$$\Updownarrow$$

$$c_1 Pr(Y=1|X=x) > c_0 Pr(Y=0|X=x)$$

$$\Updownarrow$$

$$Pr(Y=1|X=x) > \boxed{\frac{c_0}{c_1}} Pr(Y=0|X=x)$$

and set $f(x)=0$, otherwise.

Reasonable?

Yeah.

If $c_0 = c_1 \rightarrow$ Exercise 3.

If eg. $c_0 = 2c_1$, then $Pr(Y=1|X=x)$ has to be twice as large as $Pr(Y=0|X=x)$ for us to set $f(x)=1$.

Reasonable, as the related loss of false positive are twice as large as for false negatives.

Extra 7.

See pdf.