Week9-notes

Monday, 15 March 2021 12:14

Textbook

Ex 5.9

$\delta_k(x) = \log(\pi_k) - \frac{1}{2}\mu_k^T \Sigma^{-1}\mu_k + x^T\Sigma^{-1}\mu_k$

$K = 2, \quad \pi_1 = \pi_2 = 0.5$

Show $d_1(x) > d_2(x)$

$\Updownarrow$

$(\mu_1 - \mu_2)^T \Sigma^{-1}(x - \mu) > 0$

where $\mu = \frac{1}{2}(\mu_1 + \mu_2)$.

$d_1(x) - d_2(x) > 0$ (*)

$\left[-\frac{1}{2}\mu_1^T\Sigma^{-1}\mu_1 + x^T\Sigma^{-1}\mu_1\right] - \left[-\frac{1}{2}\mu_2^T\Sigma^{-1}\mu_2 + x^T\Sigma^{-1}\mu_2\right] > 0$

$[\sim\|\sim] - [\sim\|\sim] - \frac{1}{2}\mu_1^T\Sigma^{-1}\mu_2 + \frac{1}{2}\mu_1^T\Sigma^{-1}\mu_2 > 0$

$x^T\Sigma^{-1}\mu_1 - x^T\Sigma^{-1}\mu_2 - \left[+\frac{1}{2}\mu_1^T\Sigma^{-1}\mu_1 + \frac{1}{2}\mu_1^T\Sigma^{-1}\mu_2\right]$

$+ \left[+\frac{1}{2}\mu_2^T\Sigma^{-1}\mu_2 + \frac{1}{2}\mu_1^T\Sigma^{-1}\mu_2\right] > 0$

$x^T\Sigma^{-1}\mu_1 - x^T\Sigma^{-1}\mu_2 - \mu_1^T\Sigma^{-1}\underbrace{\left(\frac{1}{2}[\mu_1 + \mu_2]\right)}_{\mu}$

$+ \mu_2^T\Sigma^{-1}\left(\frac{1}{2}(\mu_1 + \mu_2)\right) >$

$x^T\Sigma^{-1}\mu_1 - x^T\Sigma^{-1}\mu_2 - \mu_1^T\Sigma^{-1}\mu + \mu_2^T\Sigma^{-1}\mu > 0$

$(\mu_1^T - \mu_2^T)\Sigma^{-1}(x - \mu) > 0$

$\mu_1^T\Sigma^{-1}x - \mu_2^T\Sigma^{-1}x - \mu_1^T\Sigma^{-1}\mu + \mu_2^T\Sigma^{-1}\mu > 0$

$(\mu_1 - \mu_2)^T\Sigma^{-1}(x - \mu) > 0$

$\underset{1\times n}{x^T}\ \underset{n\times n}{\Sigma^{-1}}\ \underset{n\times 1}{\mu_1} = \underset{1\times 1}{\text{scalar}} = \text{constant} = c$

~~Recall trace properties.~~

$c = c^T$

$x^T\Sigma^{-1}\mu_1 = (x^T\Sigma^{-1}\mu_1)^T$

$= \mu_1^T(\Sigma^{-1})^T x$

$= \mu_1^T(\Sigma^T)^{-1}x$

$= \mu_1^T\Sigma^{-1}x.$

5.10

Assume $p = 1$, $\sum_{i=1}^{n} x_i = 0$.

$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$

$y = X\beta + \varepsilon, \quad X = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix}$

what will $\hat{\beta}$ be?

$\hat{\beta} = (X^TX)^{-1}X^Ty$. assumptions

compute

$X^TX = \begin{bmatrix} \sum_{i=1}^n 1 & \sum_{i=1}^n x_i \\ \sum_{i=1}^n x_i & \sum_{i=1}^n x_i^2 \end{bmatrix} = \begin{bmatrix} n & 0 \\ 0 & \sum_{i=1}^n x_i^2 \end{bmatrix}$

$X^Ty = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$ $n_0 = n/2$, $n_1 = n/2$

$= \begin{bmatrix} n_1 \\ \sum_{i:C_i=1} x_i \end{bmatrix} = \begin{bmatrix} n_1 \\ n_1\hat{\mu}_1 \end{bmatrix}$ $(\hat{\mu}_1 = \bar{x}_1)$

$\hat{\beta} = \begin{bmatrix} n_1/n \\ n_1\hat{\mu}_1 / \sum_{i=1}^n x_i^2 \end{bmatrix}$

Following boundary

$\hat{\beta}_0 + \hat{\beta}_1 x > 1/2$

$\frac{n_1}{n} + \frac{n_1\hat{\mu}_1}{\sum_{i=1}^n x_i^2}x > 1/2$

$\frac{1}{2} + \frac{n_1\hat{\mu}_1}{\sum x_i^2}x > \frac{1}{2}$

$\underbrace{\frac{n_1\hat{\mu}_1}{\sum x_i^2}}_{\text{constant}} x > 0$

Sec 6.4.

$x > 0$

Linear discriminant analysis, $n_0 = n_1 = \frac{n}{2}$

Boundary at

$x = \hat{\mu} = \frac{1}{2}(\hat{\mu}_0 - \hat{\mu}_1) = \frac{1}{2}\left(\frac{1}{n_0}\sum_{i:C_i=0} x_i + \frac{1}{n_1}\sum_{i:C_i=1} x_i\right)$

$= \frac{1}{2\cdot\frac{n}{2}}\left(\sum_{i:C_i=0} x_i + \sum_{i:C_i=1} x_i\right)$

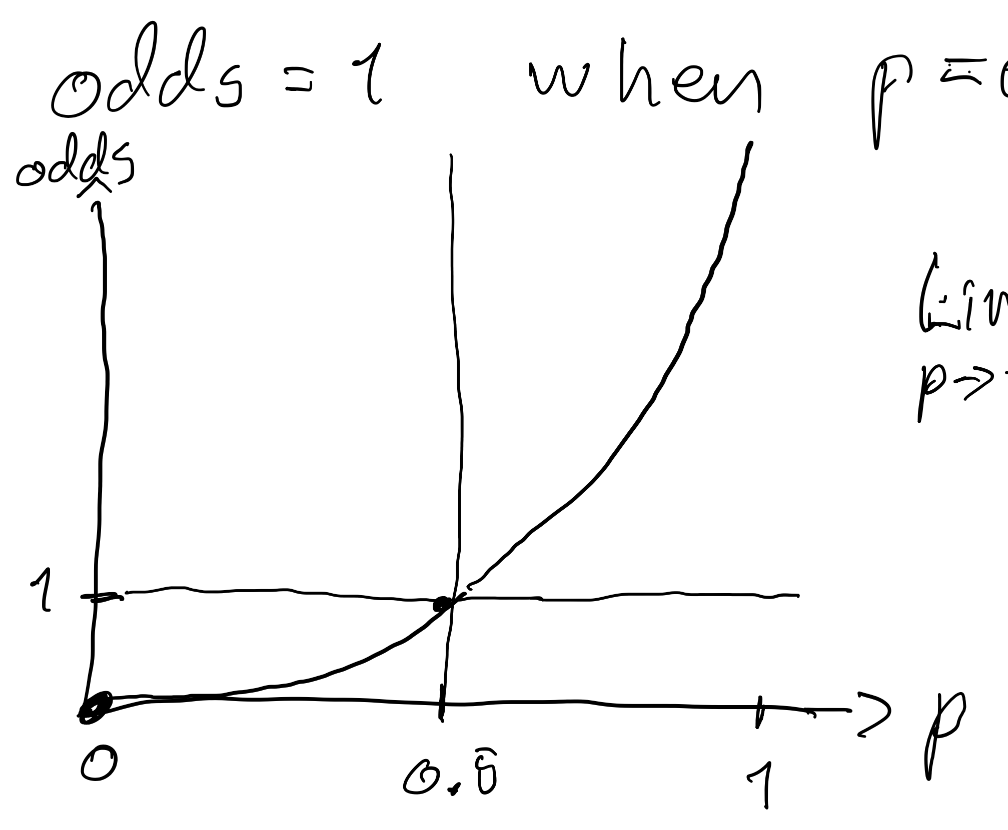
$= \frac{1}{n}\left(\sum_{i=1}^n x_i\right) = \frac{1}{n}\cdot 0 = 0.$

ISLR

5.9 odds vs probability

$\text{Odds} = \frac{p}{1-p}$

odds = 1 when $p = 0.5$



$\lim_{p\to 1} \text{odds} = \infty$

a) odds = 0.37

$\text{odds} = \frac{p}{1-p}$

$\text{odds} - p\cdot\text{odds} = p$

$\text{odds} = p(1 + \text{odds})$

$p = \frac{\text{odds}}{1 + \text{odds}} = \frac{0.37}{1.37} \approx 0.27$

b) $p = 0.16$

$\text{odds} = \frac{0.16}{0.84} \approx 0.19$

5.10 - coding.