UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in	STK2130 — Modelling by Stochastic Processes.
Day of examination:	Wednesday June 11th 2014.
Examination hours:	14.30 – 18.30.
This problem set consists of 2 pages.	
Appendices:	None.
Permitted aids:	Approved calculator. "Formelsamling" for STK1100 and STK1110

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1

A Markov chain $\{X_n, n = 0, 1, 2, \dots\}$ has state space $\{1, 2, 3, 4\}$ and transition probability matrix

$$\mathbf{P} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0.3 & 0.2 & 0.5 & 0 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0.4 & 0.6 \end{vmatrix}$$

a) Describe the Markov chain by a diagram.

b) Find all communicating classes of the Markov chain. Which are closed?

c) Which classes are recurrent and which classes are transient? Find the period of the state $i = 3$.

d) Calculate for all $i, j = 1, \dots, 4$ the probability that $X_2 = j$ given $X_0 = i$.

e) Conditioned upon the chain has entered one of the states 3 or 4 find the stationary distribution over these two states.

f) Starting in state 2 find the expected time until entering one of the recurrent states.

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g) Starting in state 2 what is the probability of ultimate absorption in the states $\{3, 4\}$?

Problem 2

a) What characterizes a birth and death process with nonnegative parameters $\{\lambda_n\}_{n=0}^{\infty}$ and $\{\mu_n\}_{n=1}^{\infty}$?

b) Assume that the limiting probabilities $\{P_n\}_{n=0}^{\infty}$ exist. Give the balance equations which are satisfied by these limiting probabilities.

c) Show that

$$\lambda_n P_n = \mu_{n+1} P_{n+1}, n = 0, 1, \ldots$$

d) Find the limiting probabilities $\{P_n\}_{n=0}^{\infty}$. Give a necessary and sufficient condition for these limiting probabilities to exist.

e) Consider the following queueing model with discouragement, where $\lambda_n = \lambda/(n+1), n = 0, 1, \ldots$ and $\mu_n = \mu, n = 1, 2, \ldots$. Why is it called a queueing model with discouragement? When does the distribution of the limiting probabilities $\{P_n\}_{n=0}^{\infty}$ for this model exist? Find it.

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