Additional exercises for STK2130 Spring 2013.

Exercise 2 Assume that $X(t)$ is a pure birth process with birth rates $\lambda_i = \lambda$ for all $X(t) = i$.

a) Argue that $X(t)$ is a Poisson-process and explain why

$$P_{ij}(t) = \frac{(\lambda t)^{(j-i)}}{(j-i)!} \exp(-\lambda t)$$

for $0 \leq i \leq j$ and $P_{ij}(t) = 0$ for $j < i$.

b) Show that these $P_{ij}(t)$ satisfy both the backward and forward Kolmogorov equations.

c) Show that the $P_{ij}(t)$ for the Poisson-process also can be generated recursively by the method in Proposition 6.4 in Ross, i.e. assume that if the formula holds for $P_{i,j-1}(t)$ then by Prop. 6.4 it also holds for $P_{ij}(t)$.

Exercise 3 Assume that we have a Yule-process, i.e. a pure birth process with birth rate $\lambda_i = i\lambda$ for $i = 1, 2, 3, \ldots$. In Ross (Example 6.8 and Exercise 5.11) it has been shown that

$P_{1j}(t) = \exp(-\lambda t)(1 - \exp(-\lambda t))^{j-1}$ for $j = 1, 2, 3, \ldots$

$P_{ij}(t) = \binom{j-1}{i-1} \exp(-i\lambda t)(1 - \exp(-\lambda t))^{j-i}$ for $i > 1$ and $j = i, i+1, i+2, \ldots$

and $P_{ij}(t) = 0$ otherwise

a) Verify that these $P_{ij}(t)$ satisfy both the backward end forward Kolmogorov equations.

b) Show that also the $P_{ij}(t)$ for the Yule-process can be generated recursively by the method in Proposition 6.4 in Ross, i.e. assume that if the formula holds for $P_{i,j-1}(t)$ then by Prop. 6.4 it also holds for $P_{ij}(t)$.

Exercise 4 A competing risk model is a simple time-continuous Markov chain with an individual starting state 0 (referred to as being alive) and eventually moving to state $k = 1, 2, ..., K$ states (referred to as dying of cause k) with a (constant) rate q_{0k}. Determine

i) $P_{00}(t) = P(X(t) = 0|X(0) = 0)$

ii) $P_{0k}(t) = P(X(t) = k|X(0) = 0)$ for $k = 1, 2, ..., K$

and calculate $\lim_{t\to\infty} P_{0k}(t)$