STK 2130

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$\{X(t),\ t \in T\}$

t := time

$X(t)$:= state

T := index set

X_t = # customers on day t

$X(t)$ = # customers at time t

T
- countable → discrete time process
- real line → continuos time process

$X(t)$

$$E[E[X \mid Y]] = E[X]$$

$$\mathrm{Var}[X] = E[\mathrm{Var}[X \mid Y]] + \mathrm{Var}[E[X \mid Y]]$$

<u>Proof</u>

$$E[E[X \mid Y]] = \int \overbrace{E[X \mid Y]}^{h(y)} f(y)\,dy$$

$$f(x,y) = f(x \mid y) f(y)$$

$$= \iint x\, f(x \mid y)\,dx\, f(y)\,dy$$

$$= \iint x\, f(x \mid y) f(y)\,dy\,dx$$

$$= \int x \int f(x,y)\,dy\,dx$$

$$= \int x\, f(x)\,dx = E[X]$$

$$\mathrm{Var}[E[X|Y]] + E[\mathrm{Var}[X|Y]] = \mathrm{Var}[X]$$

Proof

$$\mathrm{Var}[W] = E[W^2] - (E[W])^2$$

$$E[E[X^2|Y] - E[X|Y]^2] +$$

$$E[E[X|Y]^2] - (E[E[X|Y]])^2$$

$$= E[E[X^2|Y]] - \cancel{E[E[X|Y]^2]}$$

$$+ \cancel{E[E[X|Y]^2]} - E[E[X|Y]]^2$$

$$= E[X^2] - E[X]^2$$

$$= \mathrm{Var}[X]$$

$$\{X_n,\ n=0,1,2,\dots\}$$

$$\underline{P[X_{n+1}=j \mid X_n=i}, X_{n-1}=i_{n-1},\dots,X_0=0]$$

$$=P_{ij}=P[X_{n+1}=j \mid X_n=i]$$

$$0\le P_{ij}\le 1 \quad \forall i,j \qquad \sum_{j=0}^{\infty}P_{ij}=1$$

One-step transition prob. matrix

$$P=\begin{array}{c} \\ 0 \\ 1 \\ \\ \end{array}\left\|\begin{array}{ccc} 0 & 1 & \\ P_{00} & P_{01} & \dots \\ P_{10} & P_{11} & \dots \\ \vdots & & \end{array}\right\| \qquad \sum_{j=0}^{+\infty}P_{0j}=1$$

Example 4.1

X_{n+1} = tomorrow

X_n = today

$$\mathbb{P}[X_{n+1} = R \mid X_n = R] = \alpha$$

$$\mathbb{P}[X_{n+1} = R \mid X_n = \overline{R}] = \beta$$

tomorrow

today

$$P = \begin{array}{c} \\ R \\ \overline{R} \end{array} \begin{array}{c} \begin{array}{cc} R & \overline{R} \end{array} \\ \left\| \begin{array}{cc} \alpha & 1-\alpha \\ \beta & 1-\beta \end{array} \right\| \end{array} \qquad \sum_{i \in \{R, \overline{R}\}} P_{Ri} = 1$$

Example 4.3

Gary $\begin{cases} C \\ S \\ G \end{cases}$

today

$$C \begin{cases} C & 0.5 \\ S & 0.4 \\ G & 0.1 \end{cases} \qquad S \begin{cases} C & 0.3 \\ S & 0.4 \\ G & 0.3 \end{cases} \qquad G \begin{cases} C & 0.2 \\ S & 0.3 \\ G & 0.5 \end{cases}$$

$$P = \begin{array}{c} \\ C \\ S \\ G \end{array} \begin{array}{c} \begin{array}{ccc} C & S & G \end{array} \\ \left\| \begin{array}{ccc} 0.5 & 0.4 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{array} \right\| \end{array} \qquad \sum_{i \in \{C,S,G\}} p_{Ci} = 1$$

Example 4.2

state $\in \{0, 1\}$

$$P = \begin{array}{c|c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \left\| \begin{array}{cc} P_{00} & P_{01} = 1-p \\ P_{10} = 1-p & \underline{\underline{P_{11}}} \end{array} \right\| \end{array}$$