

Ex 5.45

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$$\begin{aligned} \text{Cov}(T, N(T)) &= E[(N(T) - E[N(T)])(T - E[T])] \\ &= E[N(T) \cdot T] - E[T] \cdot E[N(T)] - E[T] \cdot E[N(T)] + E[N(T)] \cdot E[T] \\ &= E[E[N(T) \cdot T | T]] - E[T] \cdot E[E[N(T) | T]] \\ &= E[T E[N(T) | T]] - E[T] \cdot E[\lambda T] \\ &= E[T \cdot \lambda T] - \lambda E[T]^2 \\ &= \lambda (E[T^2] - E[T]^2) = \lambda \sigma^2 \end{aligned}$$

$$\begin{aligned} * \text{Var}(N(T)) &= E[\text{Var}(N(T) | T)] + \text{Var}(E[N(T) | T]) \\ &= E[\lambda T] + \text{Var}(\lambda T) = \lambda \mu + \lambda^2 \sigma^2 \end{aligned}$$