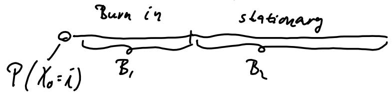
Remark 4.3.2 The chain constructed by the Hastings-Metropolis algorithm has to be started. If the starting value is not drawn from the stationary distribution π, the chain is <u>not</u> stationary. But for ergodic chains $P(X_n = j) \to \pi_j \quad j = 0, 1, 2, \ldots$

Common to have a "burn in" period until chain is stationary



feb. 18-11:14

Chapter 5. The exponential distribution and the Poisson process.

5.1 The Poisson process is a continuous time process where the distribution between jumps is exponentially distributed.

5.2 The exponential distribution

Notation $X \sim Y$ if X and Y have the same distribution. $X \sim f(x)$ means that X has the probability density function $f(x)$ and $X \sim F(x)$ if X has c.d.f. $F(X)$.

A random variable is exponentially distributed with parameter λ if the probability density function, p.d.f. is

$$f(x) = \begin{cases} \lambda \cdot e^{-\lambda x} & x \geq 0 \\ 0 & \text{else.} \end{cases}$$

The cumulative distribution function is

$$F(x) = \int_{-\infty}^{x} f(y)\,dy = \begin{cases} 1 - e^{-\lambda x} & x \geq 0 \\ 0 & \text{else} \end{cases}$$

feb. 18-11:41

The moment generating function

$$\varphi(t) = E[e^{tX}] = \int_{-\infty}^{\infty} e^{tx} \cdot \lambda \cdot e^{-\lambda x}\,dx$$

$$= \int_{-\infty}^{\infty} \lambda \cdot e^{-(\lambda - t)x}\,dx = \frac{\lambda}{\lambda - t} \quad \text{when } t < \lambda.$$

Then

$$E(X) = \frac{\partial}{\partial t}\varphi(t)\Big|_{t=0} = \frac{\lambda}{(\lambda - t)^2}\Big|_{t=0} = \frac{\lambda}{\lambda^2} = \frac{1}{\lambda}.$$

$$E(X^2) = \frac{\partial^2}{\partial t^2}\varphi(t)\Big|_{t=0} = \frac{2\lambda}{(\lambda - t)^3} = \frac{2}{\lambda^2}$$

such that $Var(X) = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$

Remark 5.2.1 The exponential distribution is much used to describe life times (times until event) in medicine and it also much in reliability

- An important property of an exponentially distributed variable is that it is <u>memoryless</u>, which means

$$P(X > s + t \mid X > t) = P(X > s)$$

feb. 18-11:52