STK2130-sp13 Problem 1

The matrix of one-step transition probabilities for a Markov chain $X_0, X_1, X_2, \ldots$ for the states $\{0, 1, 2, 3\}$ is given by

$$P = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4}. & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

a) Find the classes of the Markov chain. Which classes are transient and which are recurrent?

b) Let $P_{ij}^n = PX_n = j|X_0 = i)$. Suppose the chain starts in state $i = 1$ or state $i = 2$. Find $\lim P_{ij}^n$ when $n \to \infty$.

   What are the limits of P_{ij}^n for $i = 0$ and $i = 3$ when $n \to \infty$?

c) What is the limit for the mean of the $X's$, i.e. $\frac{1}{n}\sum_{i=1}^n X_i$, when $n \to \infty$?

   Explain your answer.

   (Remark that the limit does not depend on the initial state X_0)

d) Let T be the time until a recurrent state is reached. Find the expected value ν_j for T given that the Markov chain starts in a transient state j.

e) Let A_j be the number of visits to state j included X_0 for $j = 0$ and $j = 3$. Explain why $E[A_0|X_0 = 0] = \frac{3}{2}$ and $E[A_3|X_0 = 3] = \frac{3}{2}$.

   Also compute $E[A_0|X_0 = 0]$.