

Exercise: Let S_n denote the time of the n th event of the renewal process $\{N(t), t \geq 0\}$ having interarrival distribution F .

1. What is $P(N(t) = n | S_n = y)$?
2. Starting with

$$P(N(t) = n) = \int_0^\infty P(N(t) = n | S_n = y) f_{S_n}(y) dy$$

and using that the sum of n independent exponentials with rate λ has the Gamma (n, λ) distribution, derive $P(N(t) = n)$ when $F(y) = 1 - e^{-\lambda y}$.