$\{X_n\}$ with a state space $\mathcal{S}$

$\mathcal{A} \subset \mathcal{S}$

For each $n \geq 0$ we can define a stochastic process

$$Y_n = \mathbb{1}_{\{X_n \in \mathcal{A}\}} = \begin{cases} 1, & \text{if } X_n \in \mathcal{A} \\ 0, & \text{if } X_n \notin \mathcal{A} \end{cases}$$

$\{Y_n\}$ is a stochastic process (collection of random variables) such that for each $n \geq 0$ $Y_n \in \{0,1\}$.

The process $\{Y_n\}$ may be a Markov chain itself, but also it may <u>not</u> be a Markov chain.

(a) $\{Y_n\}$ turns out to be a Markov chain

(b) $\{Y_n\}$ turns out to not be a Markov chain

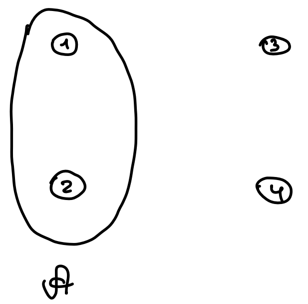
$\{Y_n\}$ is a Markov chain $\overset{\text{def}}{\iff}$ $\forall\, n, k \geq 0$ and any states $s_0, s_1, \dots, s_k, s_{k+1} \in \mathcal{S}$:

$$\mathbb{P}(Y_n = s_0, \dots, Y_{n+k} = s_k) > 0$$

$$\mathbb{P}(Y_{n+k+1} = s_{k+1} \mid Y_n = s_0, \dots, Y_{n+k} = s_k)$$
$$= \mathbb{P}(Y_{n+k+1} = s_{k+1} \mid Y_{n+k} = s_k)$$

1) Possible hint to Problem 3(b):

$\{X_n\}$



put arrows in the diagram yourself

$$\mathbb{P}(X_0 = i) = \frac{1}{4}, \quad i = 1, \dots, 4.$$

2) Hint to Problem 1(b):

1) Prove, using the definition of conditional expectation, that

$$\mathbb{E}[T \mid X_0 = i] = \sum_{j=0}^{4} \mathbb{E}[T \mid X_0 = i, X_1 = j] \cdot \mathbb{P}(X_1 = j \mid X_0 = i)$$

2) Note that
$$\mathbb{E}[T \mid X_0 = 0, X_1 = 0] = \mu_0 + 1$$
$$\mathbb{E}[T \mid X_0 = 0, X_1 = 1] = \mu_1 + 1$$
$$\mathbb{E}[T \mid X_0 = 1, X_1 = 0] = \mu_0 + 1$$
$$\mathbb{E}[T \mid X_0 = 1, X_1 = 1] = \mu_1 + 1.$$