# MODELING AIRCRAFT MOVEMENTS USING STOCHASTIC HYBRID SYSTEMS

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# Outline

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- Building and analyzing models for air traffic control in a stochastic environment
- Aircraft movements are modeled using *ordinary differential equations*
- Stochastic elements in the model:
  - Arrival of aircrafts into the system
  - Arrival and departure points of the aircrafts
- Presently the work is in a *preliminary state* where the purpose is to demonstrate how to deal with the various issues related to such models
- Ultimate goals:
  - Optimize designated aircraft trajectories w.r.t. risk
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# Main Processes and Variables

- $\mathcal{A}$ Airspace of interest
- $\{N(t)\}$ Aircraft arrival process
- $(T_{a,i}, T_{d,i})$ Arrival and departure times of $i$th aircraft
- $T_{p,i} = T_{d,i} - T_{a,i}$ Processing time of $i$th aircraft
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# Derived Processes

- $N(t) = \sum_{i=0}^{\infty} \mathrm{I}(T_{a,i} \leq t)$
- $M(t) = \sum_{i=0}^{\infty} \mathrm{I}(T_{d,i} \leq t)$
- $Q(t) = \sum_{i=1}^{\infty} \mathrm{I}(T_{a,i} \leq t < T_{d,i})$
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# Aircraft Position Model

- Aircraft trajectory:
  - $\{\boldsymbol{x}_i(t) : T_{a,i} \leq t \leq T_{d,i}\}$
- Boundary conditions:
  - $\boldsymbol{x}_i(T_{a,i}) = \boldsymbol{X}_{a,i}$
  - $\boldsymbol{x}_i(T_{d,i}) = \boldsymbol{X}_{d,i}$
- Position expressed in terms of velocity:
  - $\boldsymbol{x}_i(t) = \boldsymbol{X}_{a,i} + \int_{T_{a,i}}^{t} \dot{\boldsymbol{x}}_i(u)du$
- Assuming constant velocity (and speed $s_i$):
  - $\dot{\boldsymbol{x}}_i(t) = \frac{\boldsymbol{X}_{d,i} - \boldsymbol{X}_{a,i}}{||\boldsymbol{X}_{d,i} - \boldsymbol{X}_{a,i}||} s_i$

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# Risk Events

- $J(t)$ The index set of aircrafts present in $\mathcal{A}$ at time $t$
- The minimum distance between aircrafts at time $t$:
  - $D(t) = \min_{i,j \in J(t), i \neq j}\{\|\boldsymbol{x}_i(t) - \boldsymbol{x}_j(t)\|\}$
- $C$ Critical distance (2.5 nautical miles [4,630 meters])
- The system is in a *risky state* at time $t$ if:
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# Risk Measures

- The limiting fraction of time where the system is in a *risky state*:
  - $\bar{R} = \lim_{t\to\infty} \frac{1}{t} \int_0^t \mathrm{I}(D(u) < C)du$
- The asymptotic average minimum distance between two aircrafts in $\mathcal{A}$:
  - $\bar{D} = \lim_{t\to\infty} \frac{1}{t} \int_0^t D(u)du$
- The asymptotic average throughput:
  - $\bar{M} = \lim_{t\to\infty} \frac{M(t)}{t}$
- The asymptotic average processing time:
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# Avoiding Risk Events

## Algorithm

*For each point of time $t$ and for each $j \in J(t)$ do the following:*

STEP 1. *Calculate the "ideal" velocity for the $j$th aircraft as:*

$$\dot{\boldsymbol{x}}_j(t) = \frac{\boldsymbol{X}_{d,j} - \boldsymbol{x}_j(t)}{\|\boldsymbol{X}_{d,j} - \boldsymbol{x}_j(t)\|} s_j.$$

STEP 2. *Identify the closest aircraft, and determine whether the trajectory needs to be adjusted to avoid a risk event.*

STEP 3a. *If no action is needed, use the ideal velocity in the interval $[t, t + dt)$, and update the position.*

STEP 3b. *If an action is needed, make a "turn", i.e., replace the ideal velocity by $\dot{\boldsymbol{x}}'_j(t) = \Lambda \dot{\boldsymbol{x}}_j(t)$ in the interval $[t, t + dt)$ where $\Lambda$ is a suitable rotation matrix, and update the position.*

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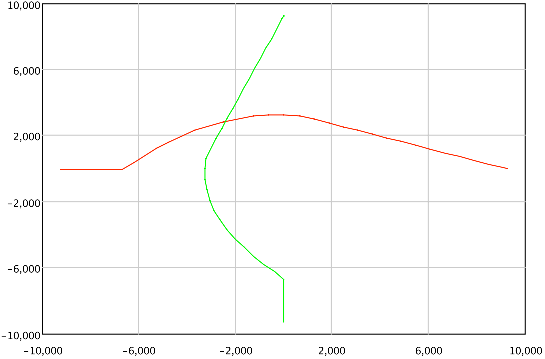
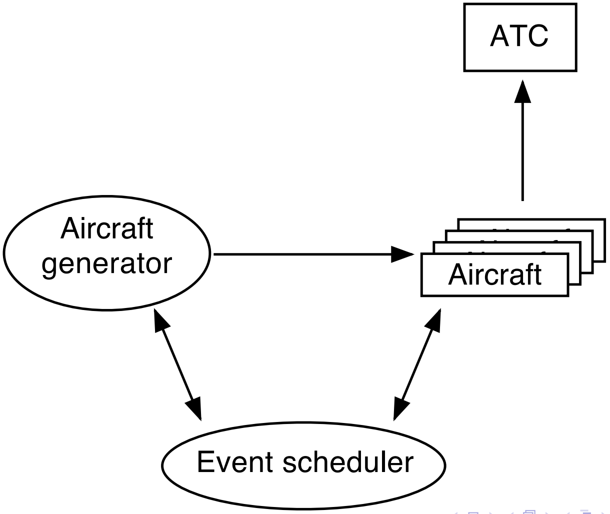
# Trajectories of two flights entering $\mathcal{A}$ at the same time



# Main Object Structure



# Hybrid Event Scheduler

- Handles both discrete events and continuous time events:
  - Aircraft generator (discrete events)
  - Aircraft position updates (continuous time events)
- Allows assigning flexible update intervals for each individual aircraft
  - Short update intervals are used in risky periods
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# The Arrival Process $\{N(t)\}$

The process $\{N(t)\}$ was chosen as a counting process with i.i.d. waiting times sampled from a *censored exponential distribution*.

Let $W_1, W_2, \ldots$ denote the waiting times between arrivals. Then $W_1, W_2, \ldots$ are sampled as:

$$W_i = \max(\kappa, U_i),\ i = 1, 2, \ldots,$$

where $U_1, U_2, \ldots$ are independent and identically exponentially distributed with mean $\mu = 90$ seconds.

The constant $\kappa$ is a control parameter limiting the traffic into $\mathcal{A}$. In the simulations $\kappa$ was varied between 10 and 50 seconds.

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# Arrival and Departure Points

- All flights through $\mathcal{A}$ are assumed to be either *northbound* or *eastbound*.
- Pr(Flight is northbound) $=$ Pr(Flight is eastbound) $= 1/2$.
- For *northbound* flights the arrival points are sampled within an interval $I_S$ of the southern border of $\mathcal{A}$ and departure points within an interval $I_N$ of the northern border of $\mathcal{A}$.
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# Simulation Results

## Without Risk Event Avoidance:

| $\kappa$ | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|
| $\bar{R}$ | 0.250 | 0.239 | 0.227 | 0.066 | 0.020 |
| $\bar{D}$ | 12,108 | 12,288 | 12,548 | 12,840 | 13,196 |
| $\bar{M}$ | 0.660 | 0.650 | 0.632 | 0.611 | 0.588 |
| $\bar{T}_p$ | 144 | 144 | 144 | 144 | 144 |

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- We have shown how to implement hybrid simulation models for aircraft trajectories in a stochastic environment
- We have analyzed two interrelated issues related to the model:
  - Managing the arrival process
  - Avoiding risk events by adjusting trajectories dynamically
- The two issues should be studied together in an integrated model
- Future work:
  - Fine-tuning the models and algorithms
  - Modeling smoother aircraft movements (in 3D)
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