UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam in:	STK 2400 - An elementary introduction to risk and reliability analysis
Date of the exam:	Thursday, December 4. 2003
Time of the exam:	0900 - 1200
The candidate is allowed to use:	Calculator

The set consists of 3 pages.

Please check that the set is complete before you start answering the questions .

In this test we consider a steering system used to control an electron beam (similar to systems used in TV sets). The system consists of five electromagnets located at the corners of an equilateral pentagon. The electron beam originates from a source behind the pentagon and passes through the pentagon near its center. A very simplified sketch of the system is shown in Figure 1.

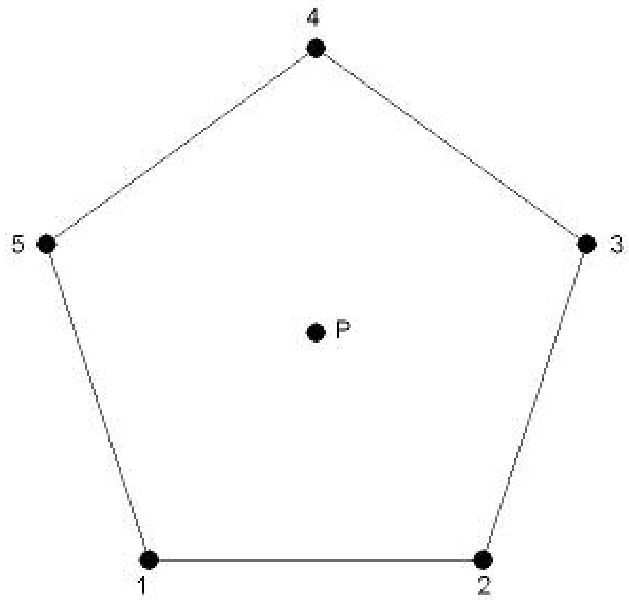

Figure 1. A system for steering an electron beam

In the figure, the components of the system, i.e., the electromagnets, are denoted by 1, … , 5, while the point where the beam passes through is denoted by *P*. It is not necessary that all the electromagnets are functioning to ensure the ability to steer the electron beam. A necessary and sufficient condition is that the point *P* is contained inside the polygon spanned by the corners of the pentagon corresponding to the functioning components. This implies that at least three components are needed to make the system work. Still, not all possible combinations of three components are sufficient. Observe e.g., that the triangle spanned by the corners 1, 2 og 3, *do not* contain P. On the other hand if components 1, 2 og 4 are working, then this is sufficient since the triangle spanned by these three corners indeed contains P.

(a) Find the minimal path sets of the system (5 sets), and the minimal cut sets of the system (5 sets as well).

(b) Show that the structure function of the system can be written as:

$$\begin{aligned}\phi(\mathbf{X}) &= X_1X_2X_4 + X_2X_3X_5 + X_1X_3X_4 + X_2X_4X_5 + X_1X_3X_5 \\ &- X_1X_2X_3X_4 - X_1X_2X_3X_5 - X_1X_2X_4X_5 - X_1X_3X_4X_5 - X_2X_3X_4X_5 \\ &+ X_1X_2X_3X_4X_5\end{aligned} \tag{1.1}$$

where $X_1, \dots, X_5$ are the component state variables.

(c) Observe that all the coefficients in the expression (1.1) are either +1 or -1. Describe another class of systems with this property.

(d) Assume at this stage that all the components are stochastically independent, and that all the components have reliability p. Show that the system reliability, h, can be written as:

$$h(p) = 5p^3 - 5p^4 + p^5 \tag{1.2}$$

(e) Assume in particular that $p = 0.75$. Compute h for this value of p. [Answer: 0.7646]

(f) Explain why the calculations in (e) implies that $h(p) > p$ if $0.75 < p < 1.0$.

(g) Make a plot of h as a function of p. Comment this plot.

In the remaining part of this test we assume that the electromagnets can fail in two ways: either the magnets burn and stop working as a result of this, or the magnets fail as a result of an electric power failure. If the electric power fails, then all the components fail. We then introduce:

$$Y_i = \begin{cases} 1 & \text{if the } i\text{-th magnet has not burned} \\ 0 & \text{otherwise} \end{cases}, \quad i = 1, \dots, 5. \tag{1.3}$$

$$Z = \begin{cases} 1 & \text{if the electric power does not fail} \\ 0 & \text{otherwise} \end{cases} \tag{1.4}$$

We assume that $Y_1, \dots, Y_5$ and Z are stochastically independent and that:

(1.5) $\Pr(Y_i = 1) = \alpha$, $i = 1, \dots, 5$, while $\Pr(Z = 1) = \theta$

(h) Explain briefly why this implies that:

(1.6) $\Pr(X_i = 1) = \alpha\theta$ $i = 1, \dots, 5$.

Are $X_1, \dots, X_5$ assosiated under these circumstances? Give a brief argument for your answer.

(i) Compute $\mathrm{Cov}(X_i, X_j)$, for $i \neq j$.

(j) Finally compute the system reliability expressed as a function of α and θ.

THE END