Exercises Ch2

Section 2.2

Exercise 2.2.1 Consider log-normal financial returns $R = e^{\xi+\sigma\varepsilon} - 1$ where $\varepsilon \sim N(0,1)$ and let $\xi = 0.005$ and $\sigma = 0.05$ (could be monthly equity). **a)** Run Monte Carlo experiments with $m = 100$, $m = 1000$ and $m = 10000$ simulations and in each case compute the average $\bar{R}^*$ and the standard deviation s^* [**R-commands:** eps=rnorm(m); R=exp(0.005+0.05*eps)-1); mean(R); sqrt(var(R)).]. **b)** Compare with the exact values $E(R) = e^{\xi+\sigma^2/2} - 1$ and $\text{sd}(R) = e^{\xi+\sigma^2/2}\sqrt{e^{\sigma^2} - 1}$. **c)** What is the prospect of determining ξ and σ if we are dealing with historical equity returns instead of simulated ones?

Exercise 2.2.2 Distributions can be compared through **Q-Q plots**. If $F^{-1}(u)$ and $G^{-1}(u)$ are the percentiles of two distribution functions $F(x)$ and $G(x)$, then $F^{-1}(u_i)$ is plotted against $G^{-1}(u_i)$ for $i = 1, \dots, n$ where $u_i = (i - 1/2)/n$. Use this technique to look at the difference between normal $R = \xi + \sigma\varepsilon$ and log-normal $R = e^{\xi+\sigma\varepsilon} - 1$ equity returns where $\varepsilon \sim N(0,1)$. **a)** Argue that $\xi + \sigma\Phi^{-1}(u)$ and $e^{\xi+\sigma\Phi^{-1}(u)} - 1$ are the precentiles of the two models when $\Phi^{-1}(u)$ is the inverse Gaussian integral. **b)** Write a program which Q-Q plots the normal against the log-normal [**R-commands:** *u=(1:n-0.5)/n; qno=ξ+σ*qnorm(u); qln=exp(qno)-1; plot(qno,qln).*]. **c)** Run the program when $n = 1000$, $\xi = 0.005$ and $\sigma = 0.05$ and try to understand the pattern. **d)** Redo c) when $\sigma = 0.005$ instead of 0.05 and note how the plot changes.

Exercise 2.2.3 Suppose $Y = a + bX$ where a and b are coefficients. If $F^{-1}(u)$ and $G^{-1}(u)$ are the pecentiles of X and Y, argue that $G^{-1}(u) = a + bF^{-1}(u)$ so that their Q-Q plot is a straight line. Changing the mean and standard deviation of a distribution only shift and rotate QQ-plots without doing anything to the shape

Exercise 2.2.4 Q-Q plotting can also be used with Monte Carlo when the underlying distribution functions are too complicated to calculate exactly. Let $X^*_{(1)} \le \dots \le X^*_{(m)}$ be ordered simulations under one model and $Y^*_{(1)} \le \dots \le Y^*_{(m)}$ under another. **a)** Explain that Q-Q plotting means $X^*_{(i)}$ being plotted against $Y^*_{(i)}$ for $i = 1, \dots, m$. **b)** Write a program which compares normal and log-normal equity returns as in Exercise 2.2.2 with Monte Carlo replacing exact mathematics [**R-commands:** *X=ξ+σ*rnorm(m); Y=exp(ξ+σ*rnorm(m))-1; plot(sort(X),sort(Y)).*]. **c)** Run the program for $\xi = 0.005$ and $\sigma = 0.05$ and 0.005. Use $m = 100000$ and verify that the conclusions in Exercise 2.2.c,d) haven't changed.

Exercise 2.2.5 a) Generate $m = 1000000$ simulations of $R = e^{\xi+\sigma\varepsilon} - 1$ when $\xi = 0.005$ and $\sigma = 0.05$ and plot the estimated density function [**R-commands:** *R=exp(ξ+σ*rnorm(m))-1; plot(density(R)).*]. This has produced an accurate reconstruction of the the exact density function, but suppose only $m = 100$ simulations could be afforded under circumstances with much heavier computation. **b)** Imitate this situation by selecting the first 100 simulations in a), plot their estimated density function and compare with the result in a) [**R-commands:** Add to those in a) *R1=R[1:100]; d1=density(R1); lines(d1$x,d1$y).*]. **c)** Redo b) when you smooth the estimate stronger [**R-commands:** *d2=density(R1,bw=0.005); lines(d2$x,d2$y).*]. **d)** Describe the errors in the estimates in b) and c). Those are typical! **e)** Experiment with other values of the smoothing parameter [**R-commands:** for example use *bw=0.01.*].

Exercise 2.2.6 a) Write a program which produces m_1 Monte Carlo evaluations of the lower

1% percentile of R of the preceding exercise with m simulations per experiment [**R-commands:** *X=ξ+σ*rnorm(m*m_1); R=matrix(exp(X)-1,m,m_1); q=apply(R,2,sort)[0.01*m].*] **b)** Let $\xi = 0.005$ and $\sigma = 0.05$ and run the program when $m = 1000$ and $m_1 = 100$. **c)** Plot the density function of the $m_1 = 100$ estimates and compute their mean which you compare with the exact value -0.105345. [**R-commands:** *plot(density(q)); mean(q).*]. **d)** Redo b)and c) when $m = 10000$ and comment on how the errors have changed. The pattern you have seen in this exercise is typical for a large variety of situations!

Exercise 2.2.7 Recall from (**??**) that $\mathrm{sd}(s^*) \doteq (\sigma/\sqrt{2m})\sqrt{1+\kappa/2}$. where κ is the kurtosis. Compute how much $\mathrm{sd}(s^*)$ is inflated when κ moves from 0 (Gaussian data) to 6 (which might be realistic for daily equity returns).

Exercise 2.2.8 The mathematical definition of kurtosis is $\kappa = E(X-\xi)^4)/\sigma^4 - 3$ where $\xi = E(X)$ and $\sigma = \mathrm{sd}(X)$, and it has a simple interpretion when X follows the stochastic volatility model (**??**). It is then assumed that $X = \xi + \sigma_\xi\sqrt{Z}\,\varepsilon$ where ε and Z are independent and $\varepsilon \sim N(0,1)$. **a)** Show that

$$(X-\xi)^2 = \sigma_\xi^2 Z\varepsilon^2 \qquad \text{so that} \qquad \sigma^2 = E(X-\xi)^2 = \sigma_\xi^2 E(Z).$$

b) By utilising that $E(\varepsilon^4) = 3$ also show that

$$(X-\xi)^4 = \sigma_\xi^4 Z^2\varepsilon^4 \qquad \text{which yields} \qquad E(X-\xi)^4 = 3\sigma_\xi^4 E(Z^2).$$

c) Deduce that $\kappa = 3\mathrm{var}(Z)/(EZ)^2$. **d)** Why is $\kappa = 0$ for normal variables? Usually Z is scaled so that $E(Z) \doteq 1$ which makes $\kappa \doteq 3\mathrm{var}(Z)$.

Exercise 2.2.9 The standard kurtosis estimate from an independent and identically distributed sample $X_1, \ldots, X_n$ is

$$\hat{\kappa} = \frac{\hat{\nu}_4}{s^4} - 3 \qquad \text{where} \qquad \hat{\nu}_4 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^4$$

and where s^2 is the ordinary sample variance. **a)** Write a program which simulates a log-normal sample $X_i = e^{\xi+\sigma\varepsilon_i}$ for $i = 1, \ldots, n$ and compute $\hat{\kappa}$.[**R-commands** *X=exp(ξ+σ*rnorm(n)); e=mean(X); $\hat{\kappa}$=mean((X-e)**4)/var(X)**2.*]. **b)** Run the program under equity return parameters $\xi = 0.005$ and $\sigma = 0.05$ and compute $\hat{\kappa}$ when $n = 100$, $n = 1000$ and $n = 10000$. **c)** Redo b) when $\xi = 0.005$ and $\sigma = 1$ (could be losses in general insurance). **d)** Compare estimation results with the exact expression which is $\kappa = e^{4\sigma^2} + 2e^{3\sigma^2} + 3e^{2\sigma^2} - 3$ (see Johnson, Kotz and Balakrishnan, 1994) and try to draw some conclusions about the kurtosis estimate.

Section 2.3
Exercise 2.3.1 The Cauchy model has density function

$$f(x) = \frac{(\beta\pi)^{-1}}{1+(x/\beta)^2} \qquad \text{and distribution function} \qquad F(x) = 1/2 + \mathrm{atan}(x/\beta)/\pi.$$

a) Show that $F^{-1}(u) = \beta\tan\{(u-1/2)\pi\}$ and write down the inversion sampler. **b)** Write a program which generates m Cachy-distributed variables. [**R-commands:** *U=runif(m); X=β*tan((U-0.5)*pi)*, or the intrinsic one *X=rcauchy(m,β)*]. **c)** Generate $m = 100$, $m = 1000$ and $m = 10000$

independent realisations when $\beta = 1$ and compute each time the mean [**R-command**: *mean(X)*.]. The pattern (or lack of it) is due to the underlying expectation being infinite. The Cauchy distribution belongs to the **NIG** family introduced in Execise 13.2.3.

Exercise 2.3.2 **Truncated** distributions which appear frequently in property insurance, are distributions confined to sub-domaines. For example, we may seek the distribution of X inside some interval (a, b). **a)** Argue that such sampling can be carried out by acceptance-rejection by generating simulations X^* from the original distribution and keeping those that satisfy $a < X^* < b$. **b)** Write a program which generates simulations from the truncated Gamma distribution [**R-commands:** *fac=ξ/α; X=fac*rgamma(m,α); X=X[X>a&X<b]*.]. **c)** Run the program with $m = 100000$, $\xi = 10$, $\alpha = 4$, $a = 5$ and $b = 15$, compute mean and standard devaitaion of the truncated distribution and find out how many simulations you have been left with [**R-commands:** *mean(X); sqrt(var(X)); length(X)*.]. **d)** Why is the standard deviation smaller than for the original distribution? Acceptance-rejection may be inefficient if the interval is short or far out from the centre. The table method of Section 4.2 is an alternative.

Exercise 2.3.3 A special case of truncation is distributions above or below some threshold, a common situation in property insurance and in finance. **a)** Modify the program in Exercise 2.3.2.b) so that only values exceeding a lower threshold a is kept [**R-commands:** *fac=ξ/α; X=fac*rgamma(m,α); X=X[X>a]*.]. **b)** Run the program when $m = 100000$, $\xi = 10$, $\alpha = 4$ and $a = 5$, 15 and 25, compute each time the mean of the simulations you keep and find out the number you have been left with [**R-commands:** *mean(X); length(X)*.]. **c)** Redo b) when $\alpha = 1$ with the other parameters as before. How has the relationship between a and the mean of the truncated distribution changed?

Exercise 2.3.4 This exercise constructs a Gamma sampler by acceptance-rejection from the exponential. Let $f(x) = cx^{\alpha-1}e^{-\alpha x}$ and $g(x) = e^{-x}$ be the two density functions where $x > 0$. **a)** Show that $f(x)/g(x)$ attains its maximum at $x = 1$ and argue that with $M = f(1)/g(1) = Ce^{-\alpha+1}$ this means that

$$\frac{f(x)}{Mg(x)} = e^{(\alpha-1)(\log(x)-x)} \leq 1.$$

b) Explain that an acceptance-rejection sampler for Gamma(α) generates uniforms U_1^* and U_2^* and enter the scheme

$$X^* \longleftarrow -\log(U_1^*) \qquad \text{which is accepted if} \qquad \log(U_2^*) < (\alpha - 1)(\log(X^*) - X^*).$$

c) Implement the algorithm [**R-commands:** *U1=runif(m); U2=runif(m) X=-log(U1), X=X[log(U2)<(a-1)*log(X)-X]*.] **d)** Run the program with $m = 100000$ and $\alpha = 2.5$ and check that average and standard deviation are close to $E(X)$=1 and sd$(X) = 1/\sqrt{2.5} \doteq 0.632$ [**R-commands:** *mean(X); sqrt(var(X))*.] **e)** Find out how many of the original sampes you have retained [**R-command:** *length(X)*.]. The exercise is an illustration of how acceptance-rejection procedures are constructed. Although the method is usable, those in the text are more efficient.

Exercise 2.3.5 Let $g(x) = e^{-x^2/2}$ with maximum at $x = 0$ so that $a = \sqrt{g(0)} = 1$ in Algorithm 2.3. **a)** Show that the minimum and maximum of $x\sqrt{g(x)}$ occurs at $x = -\sqrt{2}$ and $x = \sqrt{2}$ which means that $b_- = -e^{-1/2}/2$ and $b_+ = e^{-1/2}/2$ in Algorithm 2.3. **b)** Implement Algorithm

2.3 for the standard normal distribution [**R-commands**: *b1=-exp(-0.5)/2, b2=-b1, U1=runif(m), U2=runif(m), X=(b1+(b2-b1)*U2)/U1, X=X[U1<exp(-0.25*X*X).*]. **c)** Run it for $m = 10000$ and check that mean and standard deviation of the sample are close to 0 and 1. **d)** How many simulations did you obtain? [**R-command**: *length(X)*]. As in the previous exercise, the method is usable, yet included only to illustate the mechanics of ratio of uniforms sampling.

Section 2.4

Exercise 2.4.1 Let $X = e^{-\sigma^2/2+\sigma\varepsilon}$ where $\varepsilon \sim N(0,1)$ be a model for losses in property insurance. **a)** Draw $m = 10000$ simulations when $\sigma = 0.25$, compute their mean and plot the density function [**R-commands:** *eps=rnorm(m); X=exp(-σ**2/2+σ*eps); mean(X); plot(density(X)).*]. **b)** Redo a) when $\sigma = 0.5$ and $\sigma = 1$ and note how the density function changes while the mean does not.

Exercise 2.4.2 Consider the stochastic volatility model $R = e^{\xi+\xi_\sigma\sqrt{Z}\,\varepsilon} - 1$ for log-returns where ε and Z are independent and $\varepsilon \sim N(0,1)$. One specification for Z is $1/Z = G$ where $G \sim$ Gamma(α) which makes $\log(1+R)$ t-distributed. **a)** Generate $m = 10000$ simulations of R when $\xi = 0.005$, $\xi_\sigma = 0.08$ and $\alpha = 10$ [**R-commands:**. *eps=rnorm(m); G=rgamma(α)/α; R1=exp(ξ + ξ$_\sigma$/sqrt(G)*eps)-1,* see Exercise 2.5.1 for the sampling of G.]. **b)** Plot the density function [**R-command:** *plot(density(R1)).*].

Exercise 2.4.3 An alternative model for Z in the previous exercise could be $Z = e^{\tau^2/2+\tau\eta}$ where $\eta \sim N(0,1)$ and $\tau \geq 0$ a parameter. **a)** Use the formulae for the mean and standard deviation of log-normal variables to argue that $E(1/Z) = 1$ and $\mathrm{sd}(1/Z) = \sqrt{e^{\tau^2}-1}$. **b)** Show that $1/Z$ has the same mean and variance as G of Exercise 2.4.2 when $\tau = \sqrt{\log(1+1/\alpha)}$ so that $\tau = 0.3087$ when $\alpha = 10$ as in the previous exercise. [**Hint:** Look up sd(G) in Section 2.5.] **c)** Generate $m = 10000$ simulations of $R = e^{\xi+\xi_\sigma\sqrt{Z}\,\varepsilon} - 1$ when $\xi = 0.005$, $\xi_\sigma = 0.08$ and $\tau = 0.3087$ and plot the density function [**R-commands:** *eps=rnorm(m); eta=rnorm(m); Z=exp(τ**2/2+τ*eta); R2=exp(ξ + ξ$_\sigma$*sqrt(Z)*eps)-1; plot(density(R2)).*]. **d)** Q-Q plot the simulations against those of the previous exercise and judge how strongly the two models deviate [**R-command:** Take R1 from Exercise 2.2.2a) and use *plot(sort(R1),sort(R2)).*].

Exercise 2.4.4 Let $R_1 = e^{\xi+\sigma\varepsilon_1} - 1$ and $R_2 = e^{\xi+\sigma\varepsilon_2} - 1$ where ξ and σ are drift and volatility and $\epsilon_1, \epsilon_2 \sim N(0,1)$ with $\rho = \mathrm{cor}(\varepsilon_1, \varepsilon_2)$. **a)** Write a program which generates m simulations of $(\varepsilon_1, \varepsilon_2)$ [**R-commands:** *eps=matrix(rnorm(2*m),m,2) eps[,2]=ρ*eps[,1]+sqrt(1-ρ**2)*eps[,2].*] **b)** Extend the program in a) so that the portfolio return $\mathcal{R} = (R_1 + R_2)/2$ is simulated [**R-commands:** *R=exp(ξ+σ*eps)-1; R=0.5*(R[,1]+R[,2]).*]. Let $\xi = 0.05$ and $\sigma = 0.25$. **c)** Draw $m = 100000$ simulations of $\mathcal{R}$ when $\rho = 0$, 0.6 and 0.9, compute each time mean and standard deviation and compare their values. [**R-commands:** *mean(R); sqrt(var(R)).*].

Exercise 2.4.5 Suppose the volatilities of the previous exercise are made stochastic so that $\sigma = \xi_\sigma/\sqrt{G}$ where $G \sim$ Gamma(α). **a)** Extend the program so that it generates m simulations of $\mathcal{R}$ under this exptended model [**R-commands:** Take eps from Exercise 2.4.4a) and use *sig=ξ$_\sigma$/sqrt(rgamma(α)/α); R=exp(ξ+sig*eps)-1; R=0.5*(R[,1]+R[,2]).*]. **b)** Redo the simulations in Exercise 2.4.4c) when $\xi = 0.05$, $\xi_\sigma = 0.25$, and $\alpha = 10$ for the same values of ρ and comment on how $E(\mathcal{R})$ and sd($\mathcal{R}$) change. [**R-commands:** *mean(R); sqrt(var(R)).*].

Exercise 2.4.6 Change the equally weighted portfolio in Exercise 2.4.4 so that $\mathcal{R} = (R_1 + \ldots +$

$R_J)/J$. There are now J assets with returns $R_i = e^{\xi+\sigma\varepsilon_i} - 1$. Drift and volatility ξ and σ are the same for all as are $\rho = \text{cor}(\varepsilon_i, \varepsilon_j)$ for $i \neq j$. **a)** Write a program which generates m simulations of $(\varepsilon_1, \dots, \varepsilon_J)$ [**R-commands:** *eps=matrix(rnorm(J*m),m,J) eps[,2:J]=ρ*eps[,1]+sqrt(1-ρ**2)*eps[,2:J].*] **b)** Use the program in a) to simulate $\mathcal{R}$ [**R-commands:** *R=exp(ξ+σ*eps)-1; R=apply(R,1,mean).*]. **c)** Let $\xi = 0.05$, $\sigma = 0.25$ and $J = 5$, and draw $m = 10000$ simulations of $\mathcal{R}$ when $\rho = 0$, 0.6 and 0.9, compute each time mean and standard deviation and compare their values with those in Exercise 2.4.4c) [**R-commands:** *mean(R); sqrt(var(R)).*].

Exercise 2.4.7 An avant-garde model is to allow stochastic correlations as in Ball and Torus (2000). Again start with the model in Exercise 2.4.4, but now specify ρ as

$$\rho = \frac{(1+\rho_0)e^{\tau\eta} - (1-\rho_0)}{(1+\rho_0)e^{\tau\eta} + (1-\rho_0)} \qquad \text{where} \qquad \eta \sim N(0,1).$$

Here $\tau \geq 0$ is a parameter. **a)** How do you make ρ a fixed parameter and what's its value then? **b)** Argue that $-1 < \rho < 1$ and that ρ_0 is the median of ρ [**Hint:** The median appears when $\eta = 0$.]. **c)** Simulate$(\varepsilon_1, \varepsilon_2)$ by extending the program the in Exercise 2.4.4a) [**R-commands:** *eps=matrix(rnorm(2*m),m,2); eta=rnorm(m); Z=exp(τ*eta); rho=((1+ρ0)*Z-(1-ρ0))/((1+ρ0)*Z+ 1-ρ0); eps[,2]=ρ*eps[,1]+sqrt(1-ρ**2)*eps[,2].*] **c)** Simulate $\mathcal{R} = (R_1 + R_2)/2$ under this stochastic correlation model. [**R-commands:** Those in Exercise 2.4.4b); i.e. *R=exp(ξ+σ*eps)-1; R=0.5*(R[,1]+R[,2]).*]. **d)** Draw $m = 10000$ simulations of $\mathcal{R}$ when $\xi = 0.05$ and $\sigma = 0.25$, $\rho_0 = 0.6$ and $\tau = 0$, 0.5 and 1, compute each time mean and standard deviation and compare their values. [**R-commands:** *mean(R); sqrt(var(R)).*].

Section 2.5

Exercise 2.5.1 The way Gamma variables are defined in this book as $X = \xi G$ where G has shape α and mean 1 differs slightly from common usage where the 'standard' Gamma usually means $\xi = \alpha$. **a)** Check that public software works this way by generating $m = 1000$ simulations of G when $\alpha = 4$ and compute the sample mean [**R-commands:** *G=rgamma(1000,4)/4; mean(G).*]. **b)** Write a program producing m simulations of X [**R-commands:** *fac=ξ/α; X=fac*rgamma(m,α).*] **c)** Check that your program is correct by drawing $m = 1000$ simulations of X when $\xi = 10$ and $\alpha = 4$ and compare with the exact values $E(X) = 10$ and $\text{sd}(X) = 5$ [**R-commands:** *mean(X); sqrt(var(X)).*].

Exercise 2.5.2 Consider the Weibull model $X = \beta Y^{1/\alpha}$ where Y is exponential with mean one and α and β positive. **a)** Argue that the distribution function of X is

$$F(x) = \Pr(Y \leq (x/\beta)^\alpha) = 1 - e^{-(x/\beta)^\alpha}$$

and show that the Weibull sampler in Algorithm 2.12 is the inversion sampler. Let $\text{med}(X) = F^{-1}(1/2)$ and $\text{qd}(X) = F^{-1}(3/4) - F^{-1}(1/4)$ be median and quartile difference. **b)** Show that $\text{med}(X) \doteq \beta 0.6931^{1/\alpha}$ $\text{qd}(X) \doteq \beta(1.3862^{1/\alpha} - 0.2877^{1/\alpha})$. **c)** Write a program generating m Weibull simulations [**R-command:** *Y=rexp(m); X=β*Y**(1/α)* or use the the Weibull routine of R.]. **d)** Check the program by running it when $m = 100000$, $\alpha = 2$ and $\beta = 1$ and compute the sample median and sample quartile difference which you compare with $\text{med}(X) = 0.8326$ and $\text{qd}(X) = 0.6411$ [**R-commands:** *X=sort(X); med=X[50000]; qd=X[75000]-X[25000].*].

Exercise 2.5.3 a) Generate $m = 10000$ simulations from the Weibull distribution when $\beta = 1$ and

$\alpha = 2$, estimate the density function from the simulations and plot it [**R-commands:** *Y=rexp(m); X=β*Y**(1/α); plot(density(X)).*] **b)** Redo a) when $\alpha = 5$ and when $\alpha = 3.6$. The shape of the Weibull density varies strongly with α and appears almost symmetric for $\alpha = 3.6$. **c)** Q-Q compare with the normal when $\alpha = 3.6$ and interprete the pattern. [**R-commands:** With $m = 10000$ and X as the Weibull sample use *Y=rnorm(m); plot(sort(X),sort(Y),"l").*].

Exercise 2.5.4 The **Fréchet** distribution belongs to the so-called extreme value type and has been used as model for losses in property insurance. The distribution function is

$$F(x) = e^{-(x/\beta)^{-\alpha}}, \qquad x > 0$$

and α and β are positive parameters. **a)** Show that $F^{-1}(u) = \beta\{-\log(1-u)\}^{-1/\alpha}$. **b)** Program the inversion sampler with m simulations [**R-commands:** *U=runif(m); X=β*(-log(U))**(-1/α).*]. **c)** Run the program when $m = 100000$, $\alpha = 2$ and $\beta = 1$ and check it by comparing sample median and sample quartile difference with their exact values $\text{med}(X) = 1.2011$ and $\text{qd}(X) = 1.0151$ [**R-commands:** See Exercise 2.5.2d).].

Exercise 2.5.5 Another model for losses in property insurance is the **logistic** one for which the distribution function is

$$F(x) = 1 - \frac{1+\alpha}{1+\alpha e^{x/\beta}}, \qquad x > 0.$$

where α and β are postive parameters. **a)** Show that $F^{-1}(u) = \beta\log\{(1+u/\alpha)/(1-u)\}$. **b)** Program the inversion sampler with m simulations [**R-commands:** *U=runif(m); X=β*log((1+U/α)/(1-U)).*]. **c)** Run the program when $m = 100000$, $\alpha = 2$ and $\beta = 1$ and check it by comparing sample median and sample quartile difference with their exact values $\text{med}(X) = 0.9163$ and $\text{qd}(X) = 1.2993$ [**R-commands:** See Exercise 2.5.2d).].

Exercise 2.5.6 The **Burr** model is still another model that has been proposed for losses in property insurance. The distribution function is now

$$F(x) = 1 - \{1 + (x/\beta)^{\alpha_1}\}^{-\alpha_2}, \qquad x > 0.$$

where α_1, α_2 and β are positive parameters, **a)** Show that $F^{-1}(u) = \beta\{(1-u)^{-1/\alpha_2} - 1\}^{1/\alpha_1}$. **b)** Program the inversion sampler with m simulations [**R-commands:** *U=runif(m); X=β*(U**(-1/α₂)-1)**(1/α₁).*]. **c)** Run the program when $m = 100000$, $\alpha_1 = 2$, $\alpha_2 = 2$ and $\beta = 1$ and check it by comparing sample median and sample quartile difference with their exact values $\text{med}(X) = 0.6436$ and $\text{qd}(X) = 0.6069$ **R-commands:** See Exercise 2.5.2d).].

Exercise 2.5.7 Let $S = N_1 + N_2$ where N_1 and N_2 are independent and Poisson distributed with parameters $\lambda_1 = 4$ and $\lambda_2 = 7$. **a)** Generate $m = 1000$ simulations of S [**R-command:** *S=rpois(m,4)+rpois(m,7).*]. **b)** If $N \sim \text{Poisson}(\lambda_1 + \lambda_2)$, draw the same number of simulations of N. [**R-command:** *N=rpois(m,11).*] **c)** Compare the distributions of S and N through Q-Q plotting and comment on the pattern [**R-command:** *plot(sort(S),sort(N)).*] For the general story consult Section 8.2.

Exercise 2.5.8 Let $S = G_1 + G_2$ where $G_1 \sim \text{Gamma}(\alpha)$, $G_2 \sim \text{Gamma}(\alpha)$ and G_1 and and G_2 independent. **a)** Generate $m = 1000$ Monte Carlo samples of S when $\alpha = 2.5$ [**R-commands:**

G1=rgamma(m,2.5)/2.5; G2=rgamma(m,2.5)/2.5; S=G1+G2.]. **b)** Draw the same number of simulations of G when $G \sim$ Gamma$(2\alpha_)$ [**R-commands:** *G=rgamma(m,5)/5.*] **c)** Compare the distributions of S and G through Q-Q plotting and comment on the pattern [**R-command:** *plot(sort(S),sort(G)).*] For the general story consult Section 9.3.