1.8 Bibliographical notes

General work
Property insurance, life insurance and financial derivatives are all treated in later parts of this book. If you seek simple mathematical introductions to these subjects right away, try Straub (1998), Mikosch (2004) or Boland (2007) (property insurance), Gerber (1990) (life insurance) and Roman (2003) or Benth (2004) (financial derivatives).

Monte Carlo and implementation
The main theme of the present chapter has been Monte Carlo as a problem solver. Introductory books emphasizing the role of the computer are scarce in insurance (Daykin, Pentikäinen and Pesonen (1994) is an exception), but there are more of them in finance, for example Shaw (1998), Benninga (2001) and Evans and Olson (2002). Coding and implementation is a fairly young scientific discipline, but old enough for reviews on how it's done having started to appear in computer science. Hanson (1999) discusses program *re-use* and Baier and Katoen (2008) treats program *verification* with stochastic models included; see also Kaner, Falk and Ngyuen (1999). These are themes that may may merit more attention that has been provided here. Section 3.3 gave examples where output was tested against known mathematical formulae for special cases. This is often a helpful type of technique.

Other numerical methods
Compound distributions of portfolio liabilities (Section 3.3) and ruin probabilities (Section 3.6) have often been tackled by methods other than Monte Carlo. Simple approximations coming from the central limit theorem and its Cornish-Fisher extension will be presented in Section 10.2. So-called **saddlepoint** approximations is another possibility (see Jensen, 1995). Very popular in certain quarters is the **Panjer** recursion which works with discrete claim size distributions. A continuous distribution is always *approximately* discrete (see Section 4.2), and the discretization is no limitation on practical use. The approach was popularized by Panjer (1981) following earlier work by Adelson (1966). Dickson (2005) and especially Sundt and Vernic (2009) are reviews. The original idea has been extended to cover ruin too; see Dickson (2005) for an outline, but financial risk is *not* included, and Panjer recursions lack the versatility of Monte Carlo.

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1.9 Exercises

Section 3.2

Exercise 3.2.1 Let $\mathcal{X}$ be the portfolio loss when there are J identical risks. If the numbers of claims $\mathcal{N} \sim \text{Poisson}(J\mu T)$ and $\xi = E(Z)$ and $\sigma = \text{sd}(Z)$ apply per incident, then $E(\mathcal{X}) = J\mu T\xi$ and $\text{var}(\mathcal{X}) = J\mu T(\sigma^2 + \xi^2)$ which will be proved in Section 6.3. **a)** Verify by means of these results that

$$\frac{\text{sd}(\mathcal{X})}{E(\mathcal{X})} = \sqrt{\frac{1 + \sigma^2/\xi^2}{J\mu T}}.$$

b) In what sense are branches with rare events the most risky ones?

Exercise 3.2.2 a) If Z in the previous exercise is Gamma-distributed so that $\sigma = \xi/\sqrt{\alpha}$, show that $\text{sd}(\mathcal{X})/E(\mathcal{X}) = \sqrt{(1 + 1/\alpha)/(J\mu T)}$. **b)** The same question for the Pareto-distribution for which $\sigma = \xi\sqrt{\alpha/(\alpha-2)}$ for $\alpha > 2$ (Section 2.5); i.e. show that $\text{sd}(\mathcal{X})/E(\mathcal{X}) = \sqrt{(\alpha-1)/\{(\alpha-2)J\mu T\}}$. **c)** What is the rationale behind the ratios in a) and b) being decreasing functions of α?

Exercise 3.2.3 Proportional re-insurance means that liabilities are split between cedent and re-insurer in fixed ratios. If the contract applies per incident, the re-insurer obligation is $Z^{\text{re}} = \gamma Z$ where $0 < \gamma < 1$. **a)** Argue that the same relationship $\mathcal{X}^{\text{re}} = \gamma\mathcal{X}$ applies on portfolio level. **b)** Show that the relative risk $\text{sd}(\mathcal{X}^{\text{re}})/E(\mathcal{X}^{\text{re}})$ is independent of γ and the same as for the cedent.

Exercise 3.2.4 Let $Z^{\text{re}} = \min(Z - a, 0)$ be re-insurance per event and assume Pareto-distributed losses with parameters α and β. **a)** How do you compute the pure re-insurance premium when the number of claims is Poisson-distributed with parameter $\lambda = \mu T$? [**R-commands:** *U=runif(m); Z=*β**(U**(-1/*α*)-1); Z*$^{\text{re}}$*=pmax(Z-a,0);* π^{re}*=*λ**mean(Z* $^{\text{re}}$*)*]. A closed formula is another possibility; see Section 10.6.]. **b)** Compute the pure re-insurance premium for $a = 0$, 1, 4 and 6 when $\lambda = 10$, $\alpha = 3$ and $\beta = 2$ using $m = 1000000$ simulations. **c)** Redo a couple of times to look at the Monte Carlo uncertainty.

Exercise 3.2.5 Re-insurers often transfer risk to other re-insurers. For contracts per event the situation is:

Z	$\longrightarrow$	$Z_1^{\text{re}} = H_1(Z)$	$\longrightarrow$	$Z_2^{\text{re}} = H_2(Z_1^{\text{re}})$
cedent		*first reinsurer*		*second re-insurer*

Suppose $H_1(z) = \max(z - a, 0)$ and $H_2(z) = \gamma\max(z - b, 0)$ with all other conditions as in the previous exercise. **a)** How do you compute the pure premium of the second re-insurance? [**R-commands:** *U=runif(m);*