Solutions

→ (*)

1) a) Min. path sets: $\{1,4\}, \{1,3,5\}, \{2,5\}, \{2,3,4\}$

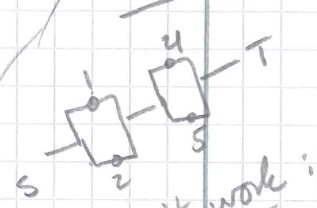
Min. cut sets: $\{1,2\}, \{1,3,5\}, \{4,5\}, \{2,3,4\}$

b) Use the factorization algorithm, and pivot wrt. component 3:

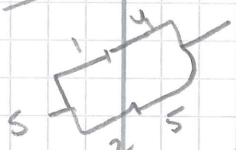
$$h_\phi(\vec{p}) = p_3 h_\phi(1_3, \vec{p}) + (1-p_3) h_\phi(0_3, \vec{p})$$

$$= p_3 (p_1 \amalg p_2)(p_4 \amalg p_5) + (1-p_3)(p_1 p_4 \amalg p_2 p_5)$$

3 works:



3 doesn't work:



$$= p_3 (p_1 + p_2 - p_1 p_2)(p_4 + p_5 - p_4 p_5) + (1-p_3)(p_1 p_4 + p_2 p_5 - p_1 p_2 p_4 p_5)$$

c) $\vec{X}^D = (X_1^D, \ldots, X_5^D) = (1-X_1, \ldots, 1-X_5)$

$$\phi^D(\vec{X}^D) = (X_1^D \cdot X_2^D) \amalg (X_1^D \cdot X_3^D \cdot X_5^D) \amalg (X_4^D \cdot X_5^D) \amalg (X_2^D \cdot X_3^D \cdot X_4^D)$$

min. cut sets for ϕ are min. path sets for ϕ^D; structure function is parallell of min. path series structures

We see that this corresponds to a bridge structure (by renaming the components; Compare to (*)!):

Φ^D: