### Introduction to Riscue

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Given *n* random variables  $X_1, \ldots, X_n$  with known probability distributions.

Let *Y* be a random variable such that:

$$Y = f(X_1,\ldots,X_n)$$

The distribution of *Y* can be difficult to calculate analytically.

Instead we run a Monte Carlo simulation where we generate *N* samples from the distributions of the  $X_i$ 's, where *N* is a large number (e.g., N = 10000).

### Monte Carlo simulation (cont.)

Monte Carlo samples:

$$Y_{1} = f(X_{1,1}, \dots, X_{n,1}),$$
  

$$Y_{2} = f(X_{1,2}, \dots, X_{n,2}),$$
  

$$\dots$$
  

$$Y_{N} = f(X_{1,N}, \dots, X_{n,N})$$

The distribution of *Y* is the estimated using the empirical cumulative distribution function, often referred to as the S-curve:

$$\hat{F}(y) = \frac{1}{N} \sum_{j=1}^{N} I(Y_j \leq y)$$

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### Monte Carlo simulation (cont.)

Note that for a given *y* we have:

$$E[\hat{F}(y)] = \frac{1}{N} \sum_{j=1}^{N} E[I(Y_j \le y)] = \frac{1}{N} \sum_{j=1}^{N} P(Y \le y)$$
$$= P(Y \le y) = F(y)$$
$$/ar[\hat{F}(y)] = \frac{1}{N^2} \sum_{j=1}^{N} Var[I(Y_j \le y)] = \frac{1}{N^2} \sum_{j=1}^{N} P(Y \le y)[1 - P(Y \le y)]$$
$$= \frac{1}{N} P(Y \le y)[1 - P(Y \le y)] \le \frac{1}{4N}$$

Thus,  $\hat{F}(y)$  is an unbiased and consistent estimator for F(y).



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### Monte Carlo simulation (cont.)



Figure: An empirical distribution curve

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- Random variables are represented by nodes in a graph
- Relations between the variables are represented by directed edges
- Large complex models can be structured hierarchically
- Simulations can be run very fast
- Results can be plotted and analyzed
- Sensitivity analysis can be integrated in the models

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### Figure: A Riscue<sup>™</sup> model

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### Application areas

### General risk models

- Cost risk models
- Schedule risk models
- Integrated risk models
- Oil production models
- Reliability models
- Insurance models

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