STK3405 - Case study: Boat engines

A. B. Huseby & K. R. Dahl

Department of Mathematics
University of Oslo, Norway

Section 9.1: Case study of fishing boat engines

Reliability analysis and comparison of two different engines for fishing boats.

We say that a boat engine is functioning if and only if the propeller and the power supply are functioning.

First engine

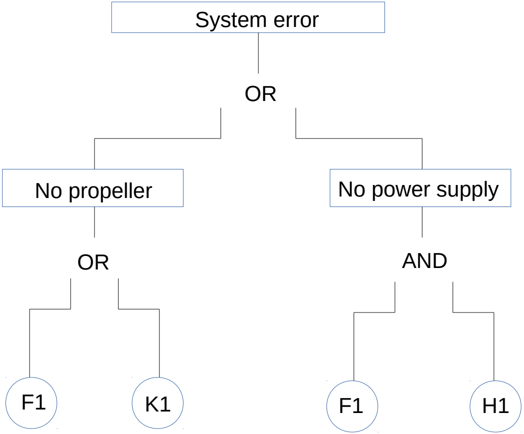
The critical parts of this engine are a propulsion engine, an auxiliary engine and a hydraulic operated clutch.

Define the following fault-events which may happen in this engine:

$$
\begin{aligned}
F1 &: \text{The propulsion engine fails} \\
K1 &: \text{The hydraulic clutch fails} \\
H1 &: \text{The auxiliary engine fails}
\end{aligned} \tag{1}
$$

Fault tree: Engine 1

Corresponding to the engine, we can make a fault tree to illustrate what may cause engine failure:



Run time for the components

The run time for the propulsion engine and the clutch is 3000 hours (per year).

The run time for the auxiliary engine is 2000 hours (per year).

We assume that the lifetime distributions of the components are exponential.

This may be unrealistic due to the memoryless property of the exponential distribution $\rightarrow$ The Weibull distribution (takes ageing of components into account) would be better.

Point estimates for the failure rates

Point estimates for the failure rates, measured in failures per hour run are:

For a diesel engine (such as the propulsion engine and the auxiliary engine), this failure rate is estimated to be $2.96 \cdot 10^{-4}$.

For a magnetic clutch, the estimate is $3.88 \cdot 10^{-5}$, while for a mechanic clutch it is $6.0 \cdot 10^{-6}$.

Due to lack of data for hydraulic clutches, the error estimate for the magnetic clutch was used as an approximation.

Probabilities of the fault events

We can compute the estimates for the probability of the basic fault events happening in one year:

$$
\begin{aligned}
P(F1) &= 1 - \exp(-2.96 \cdot 10^{-4} \cdot 3000) = 0.58852, \\
P(K1) &= 1 - \exp(-3.88 \cdot 10^{-5} \cdot 3000) = 0.10988, \\
P(H1) &= 1 - \exp(-2.96 \cdot 10^{-4} \cdot 2000) = 0.44678.
\end{aligned}
\tag{2}
$$

This gives the following estimates:

$$
\begin{aligned}
P(\text{propeller failure in a year}) &= 1 - (1 - P(F1))(1 - P(K1)) = 0.63373, \\
P(\text{no power supply in a year}) &= P(H1) \cdot P(F1) = 0.26294.
\end{aligned}
$$

Incorrect approach

From this, Fiskeriteknologisk Forskningsinstitutt concluded that:

$$P(\text{system failure in a year}) = 1-(1-0.63373)(1-0.26294) = 0.73004. \quad (3)$$

However, this method is incorrect because it does not take into account the dependence which the basic fault $F1$ (propulsion engine failure) creates in the fault tree because it occurs twice.

Correct approach

To derive the correct reliability (or equivalently, the probability of system failure within a year), we introduce the following binary variables:

$$X_1 = \begin{cases} 1 & \text{if } F1 \text{ does } \textit{not} \text{ occur within a year} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

$$X_2 = \begin{cases} 1 & \text{if } K1 \text{ does } \textit{not} \text{ occur within a year} \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

Introducing binary variables

$$X_3 = \begin{cases} 1 & \text{if } H1 \text{ does } \textit{not} \text{ occur within a year} \\ 0 & \text{otherwise} \end{cases}$$

The fault tree 1 is equivalent to the reliability block diagram below (comp. 1 corresponds to the propulsion engine, comp. 2 corresponds to the clutch, while comp. 3 corresponds to the auxiliary engine.).

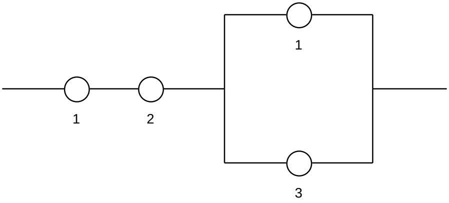


Figure: System equivalent to the fault tree.

The structure function of the engine

From Figure 2, we see that component 3 (the auxiliary engine) is an irrelevant component.

This can also be seen by deriving the structure function of the system in Figure 2:

$$
\begin{aligned}
\phi_1(\mathbf{X}) &= X_1 X_2 (1 - (1 - X_1)(1 - X_3)) \\
&= X_1 X_2 + X_1 X_2 X_3 - X_1 X_2 X_3 \\
&= X_1 X_2.
\end{aligned}
$$

The correct probability of system error

Hence, we get the following correct estimate of the probability of system error within a year.

$$
\begin{aligned}
&P(\text{System failure within a year}) \qquad (6)\\
&= P(\phi_1(\mathbf{X}) = 0) = 1 - P(\phi_1(\mathbf{X}) = 1)\\
&= 1 - (1 - P(F1))(1 - P(K1)) = 0.63373
\end{aligned}
$$

By comparing this to the estimate in equation (3), we see that Fiskeriteknologisk Forskningsinstutt ended up with a failure probability which is larger than the actual one.

Engine 2

Now consider a two-engine system consisting of right and left propulsion engines, right and left hydraulic clutches and a mechanical clutch.

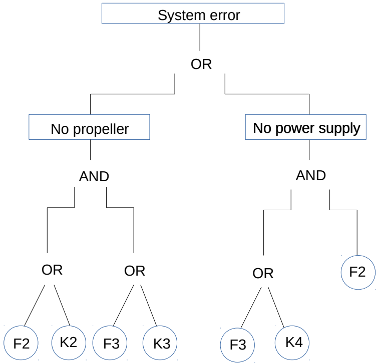
We define the following basic fault-events:

$$\begin{aligned} F2 &: \text{ Error in the left propulsion engine} \\ K2 &: \text{ Error on left hydraulic clutch} \\ F3 &: \text{ Error in the right propulsion engine} \\ K3 &: \text{ Error on right hydraulic clutch} \\ K4 &: \text{ Error on mechanical clutch} \end{aligned} \tag{7}$$

Run time per year

The run time per year for the right engine is 5000 hours, and for the mechanical clutch it is 2000 hours (only used when docking).

For the left propulsion engine and the two hydraulic clutches, the run time per year is 3000 hours (only used at sea).



Probabilities for the basic fault events

Based on the point estimates for the failure probabilities, we compute the following estimates for the probabilities of the basic fault-events:

$$
\begin{aligned}
P(F2) &= P(F1) = 0.58852, \\
P(K2) &= P(K3) = P(K1) = 0.10988, \\
P(F3) &= 1 - \exp(-2.96 \cdot 10^{-4} \cdot 5000) = 0.77236, \\
P(K4) &= 1 - \exp(-6 \cdot 10^{-6} \cdot 2000) = 0.01192.
\end{aligned} \tag{8}
$$

Estimates of failure events

This leads to the following estimates:

$$\begin{aligned}
&P(\text{propeller failure in a year}) \\
&\quad = [1-(1-P(F2))(1-P(K2))][1-(1-P(F3))(1-P(K3))] \\
&\quad = 0.63372 \cdot 0.79737 \\
&\quad = 0.50532,
\end{aligned}$$

$$\begin{aligned}
&P(\text{no power supply in a year}) \\
&\quad = [1-(1-P(F3))(1-P(K4))]P(F2) \\
&\quad = 0.77508 \cdot 0.58852 \\
&\quad = 0.45615.
\end{aligned}$$

Incorrect approach

If we make the same kind of error as was done for the one-engine system in equation (3), we find that:

$$P(\text{system error in a year}) = 1 - (1 - 0.50532)(1 - 0.45615) = 0.73097. \tag{9}$$

Correct approach

To correct this, introduce binary random variables:

$$X_1 = \begin{cases} 1 & \text{if } F2 \text{ does not occur within a year} \\ 0 & \text{if } F2 \text{ occurs within a year} \end{cases} \tag{10}$$

$$X_2 = \begin{cases} 1 & \text{if } K2 \text{ does not occur within a year} \\ 0 & \text{if } K2 \text{ occurs within a year} \end{cases}$$

(11)

Binary random variables

$$X_3 = \begin{cases} 1 & \text{if } F3 \text{ does not occur within a year} \\ 0 & \text{if } F3 \text{ occurs within a year.} \end{cases}$$

$$X_4 = \begin{cases} 1 & \text{if } K3 \text{ does not occur within a year} \\ 0 & \text{if } K3 \text{ occurs within a year.} \end{cases}$$

$$X_5 = \begin{cases} 1 & \text{if } K4 \text{ does not occur within a year} \\ 0 & \text{if } K4 \text{ occurs within a year.} \end{cases}$$

Reliability block diagram

Let component 1 correspond to the left propulsion engine, component 2 correspond to the left hydraulic clutch and so on, the fault tree for the two-engine system is equivalent to the reliability block diagram shown below.

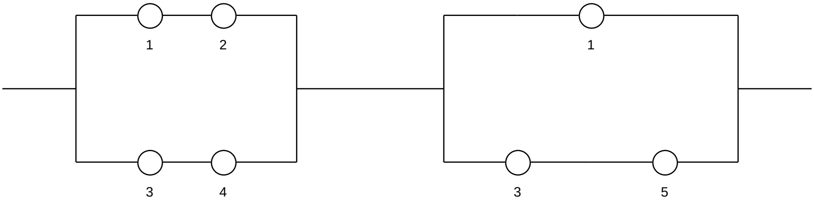


Figure: System equivalent to the fault tree for the two-engine system.

The structure function

From this, we get that the structure function for the two-engine system is:

$$
\begin{aligned}
\phi_2(\mathbf{X}) &= [1-(1-X_1X_2)(1-X_3X_4)][1-(1-X_1)(1-X_3X_5)] \\
&= (X_1X_2 + X_3X_4 - X_1X_2X_3X_4)(X_1 + X_3X_5 - X_1X_3X_5) \\
&= X_1X_2 + X_1X_3X_4 - X_1X_2X_3X_4 + X_1X_2X_3X_5 + X_3X_4X_5 \\
&- X_1X_2X_3X_4X_5 - X_1X_2X_3X_5 - X_1X_3X_4X_5 + X_1X_2X_3X_4X_5 \\
&= X_1X_2 + X_1X_3X_4 - X_1X_2X_3X_4 + X_3X_4X_5 - X_1X_3X_4X_5 \\
&= X_1X_2 + X_3X_4[X_1(1-X_2) + X_5(1-X_1)].
\end{aligned}
$$

Hence, all of the components are relevant since they are all part of the structure function.

The correct failure probability estimate

From this, we derive the correct estimate for the probability of the two-engine system failing in a year:

$$
\begin{aligned}
&P(\text{System error within a year}) \qquad (12)\\
&= 1 - P(\phi_2(\mathbf{X}) = 1)\\
&= 1 - E[\phi_2(\mathbf{X})]\\
&= 1 - \big((1 - P(F2)) \cdot (1 - P(K2)) + (1 - P(F3)) \cdot (1 - P(K3))\\
&\cdot [(1 - P(F2))P(K2) + (1 - P(K4))P(F2)]\big)\\
&= 0.50666.
\end{aligned}
$$

Conclusion

If we are not careful with taking the system structure into account in the correct way, the two systems are (almost) equally reliable.

If we do the reliability analysis in the correct way, considering the structure of the system, we find from equations (6) and (12) that the two-engine system is actually more reliable than the one-engine system (which was the system that was approved by Fiskeriteknologisk Forskningsinstitutt).

Intuitively, this is actually quite obvious, since in the two-engine system, both engines can be used for both running the propeller and supplying power. In the one-engine system, the auxiliary engine is an irrelevant component.