STK3405 – Lecture 13

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Case study: Boat engines

Reliability analysis and comparison of two different engines for fishing boats.

We say that a boat engine is functioning if and only if the propeller and the power supply are functioning.

First engine

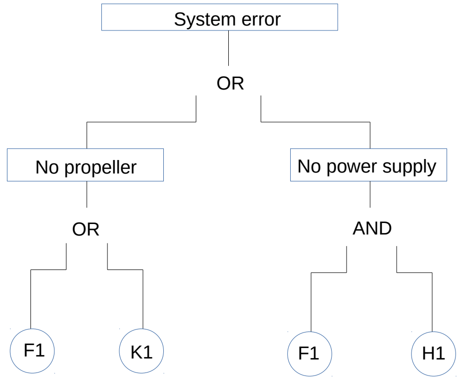
The critical parts of this engine are a propulsion engine, an auxiliary engine and a hydraulic operated clutch.

We define the following fault-events which may happen in this engine:

$$
\begin{aligned}
F_1 &= \text{The propulsion engine fails} \\
K_1 &= \text{The hydraulic clutch fails} \\
H_1 &= \text{The auxiliary engine fails}
\end{aligned}
$$

Fault tree: Engine 1

Corresponding to the engine, we can make a fault tree to illustrate what may cause engine failure:



Usage times for the components

The usage time for the propulsion engine and the clutch is 3000 hours per year.

The usage time for the auxiliary engine is 2000 hours per year.

We assume that the lifetime distributions of the components are exponential.

This may be unrealistic due to the memoryless property of the exponential distribution. A Weibull distribution which also takes ageing of components into account) might be better.

Point estimates for the failure rates

Point estimates for the failure rates, measured in failures per hour usage are:

- Propulsion engine: $\lambda_{PE} = 2.96 \cdot 10^{-4}$
- Auxiliary engine: $\lambda_{AE} = 2.96 \cdot 10^{-4}$
- Magnetic clutch: $\lambda_{MAGN} = 3.88 \cdot 10^{-5}$
- Mechanic clutch: $\lambda_{MECH} = 6.00 \cdot 10^{-6}$
- Hydraulic clutch[1] : $\lambda_{HYDR} = 3.88 \cdot 10^{-5}$

[1] Due to lack of data the estimated failure rate for the magnetic clutch was used for the hydraulic clutch as well

Probabilities of the fault events

Using the estimated failure rates we can compute the estimates for the probability of the basic fault events happening in one year:

$$P(F_1) = 1 - \exp(-\lambda_{PE} \cdot 3000) = 0.58852$$

$$P(K_1) = 1 - \exp(-\lambda_{HYDR} \cdot 3000) = 0.10988$$

$$P(H_1) = 1 - \exp(-\lambda_{AE} \cdot 3000) = 0.44678$$

Hence, we get:

$$P(\text{Propeller failure}) = 1 - (1 - P(F_1))(1 - P(K_1)) = 0.63373$$

$$P(\text{Power failure}) = P(H_1) \cdot P(F_1) = 0.26294$$

Incorrect approach

From this, the ship certifier concluded incorrectly that:

$$P(\text{system failure}) = 1 - (1 - 0.63373) \cdot (1 - 0.26294) = 0.73004$$

However, this method is incorrect because it does not take into account the dependence which the basic fault event F_1 (propulsion engine failure) creates in the fault tree because it occurs twice.

Correct approach

We recall the failure events:

$$
\begin{aligned}
F_1 &= \text{The propulsion engine fails} \\
K_1 &= \text{The hydraulic clutch fails} \\
H_1 &= \text{The auxiliary engine fails}
\end{aligned}
$$

To obtain the correct result we introduce the following binary variables:

$$
\begin{aligned}
X_1 &= I(F_1 \text{ does not occur within a year}) \\
X_2 &= I(K_1 \text{ does not occur within a year}) \\
X_3 &= I(H_1 \text{ does not occur within a year})
\end{aligned}
$$

We also let $p_i = P(X_i = 1)$, $i = 1, 2, 3$. Thus we have:

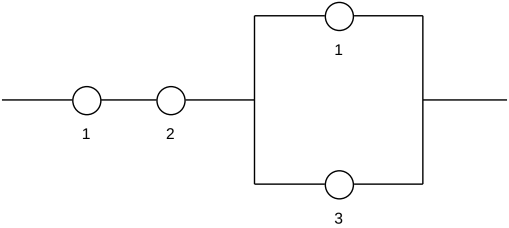
$$
\begin{aligned}
p_1 &= 1 - P(F_1) = 1 - 0.58852 = 0.41148 \\
p_2 &= 1 - P(K_1) = 1 - 0.10988 = 0.89012 \\
p_3 &= 1 - P(H_1) = 1 - 0.44678 = 0.55322
\end{aligned}
$$

Reliability block diagram

We note that the event that a system failure does not occur is equivalent with that

- A propeller failure does not occur
- A power failure does not occur

Thus, the system can be represented by the following reliability block diagram:



The structure function of the engine

From the reliability block diagram, it is easy to see that Component 3 (the auxiliary engine) is an irrelevant component.

This can also be seen by deriving the structure function of the system:

$$\begin{aligned}\phi_1(\boldsymbol{X}) &= X_1 X_2 [X_1 \amalg X_3] \\ &= X_1 X_2 [X_1 + X_3 - X_1 X_3] \\ &= X_1 X_2 + X_1 X_2 X_3 - X_1 X_2 X_3 \\ &= X_1 X_2\end{aligned}$$

In this expression we have introduced $\boldsymbol{X} = (X_1, X_2, X_3)$.

The correct probability of system error

Hence, we get the following correct estimate of the probability of system error within a year:

$$
\begin{aligned}
&P(\text{System failure within a year}) \\
&= P(\phi_1(\boldsymbol{X}) = 0) = 1 - P(\phi_1(\boldsymbol{X}) = 1) \\
&= 1 - p_1 p_2 \\
&= 1 - 0.41148 \cdot 0.89012 = 0.63373
\end{aligned}
$$

By comparing this to the incorrect probability, 0.73004, we see that certifier ended up with a failure probability which is significantly larger than the correct probability.

Engine 2

Now consider a two-engine system consisting of right and left propulsion engines, right and left hydraulic clutches and a mechanical clutch.

We define the following basic fault-events:

$$F_2 = \text{The left propulsion engine fails}$$

$$K_2 = \text{The left hydraulic clutch fails}$$

$$F_3 = \text{The right propulsion engine fails}$$

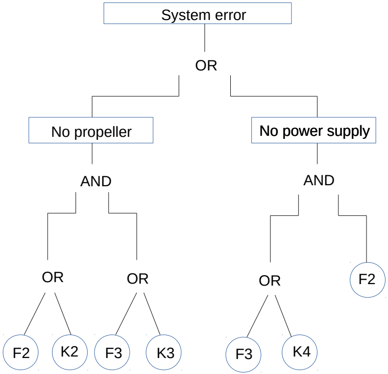
$$K_3 = \text{The right hydraulic clutch fails}$$

$$K_4 = \text{The mechanical clutch fails}$$

Usage time per year

The run time per year for the right engine is 5000 hours, and for the mechanical clutch it is 2000 hours (only used when docking).

For the left propulsion engine and the two hydraulic clutches, the run time per year is 3000 hours (only used at sea).



Probabilities for the basic fault events

Based on the point estimates for the failure rates, we compute the following estimates for the probabilities of the basic fault-events:

$$P(F_2) = P(F_1) = 0.58852$$

$$P(K_2) = P(K_3) = P(K_1) = 0.10988$$

$$P(F_3) = 1 - exp(-\lambda_{PE} \cdot 5000) = 0.77236$$

$$P(K_4) = 1 - exp(-\lambda_{MECH} \cdot 2000) = 0.01192.$$

Estimates of failure events

This leads to the following estimates:

$$
\begin{aligned}
&P(\text{Propeller failure}) \\
&\quad = [1 - (1 - P(F_2))(1 - P(K_2))][1 - (1 - P(F_3))(1 - P(K_3))] \\
&\quad = 0.63372 \cdot 0.79737 = 0.50532
\end{aligned}
$$

$$
\begin{aligned}
&P(\text{Power failure}) \\
&\quad = [1 - (1 - P(F_3))(1 - P(K_4))] \cdot P(F_2) \\
&\quad = 0.77508 \cdot 0.58852 = 0.45615
\end{aligned}
$$

Incorrect approach

If we make the same kind of error as was done for the one-engine system, we would find that:

$$
\begin{aligned}
&P(\text{System failure}) \\
&\quad = 1 - [1 - P(\text{Propeller failure})] \cdot [1 - P(\text{Power failure})] \\
&\quad = 1 - [1 - 0.50532] \cdot [1 - 0.45615] = 0.73097
\end{aligned}
$$

NOTE: With the same incorrect approach we found that the probability of system failure for Engine 1 was 0.73004. Thus, using this approach Engine 2 appears to have a (slightly) higher failure probability than Engine 1!!

Correct approach

To obtain the correct result we again introduce the following binary variables:

$$X_1 = I(F_2 \text{ does not occur within a year})$$
$$X_2 = I(K_2 \text{ does not occur within a year})$$
$$X_3 = I(F_3 \text{ does not occur within a year})$$
$$X_4 = I(K_3 \text{ does not occur within a year})$$
$$X_5 = I(K_4 \text{ does not occur within a year})$$

We also let $p_i = P(X_i = 1)$, $i = 1, 2, 3, 4, 5$. Thus we have:

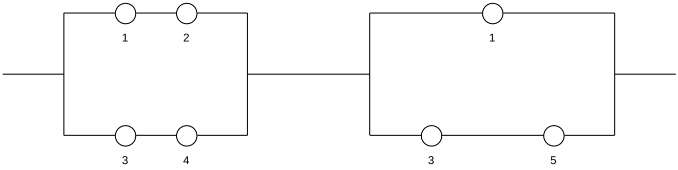
$$p_1 = 1 - P(F_2) = 1 - 0.58852 = 0.41148$$
$$p_2 = 1 - P(K_2) = 1 - 0.10988 = 0.89012$$
$$p_3 = 1 - P(F_3) = 1 - 0.77236 = 0.22764$$
$$p_4 = 1 - P(K_3) = 1 - 0.10988 = 0.89012$$
$$p_5 = 1 - P(K_4) = 1 - 0.01192 = 0.98808$$

Reliability block diagram

We note that the event that a system failure does not occur is equivalent with that

- A propeller failure does not occur
- A power failure does not occur

Thus, the system can be represented by the following reliability block diagram:



The structure function

The structure function of this system is:

$$\phi_2(\boldsymbol{X}) = [(X_1 \cdot X_2) \amalg (X_3 \cdot X_4)] \cdot [X_1 \amalg (X_3 \cdot X_5)]$$

$$= [X_1 X_2 + X_3 X_4 - X_1 X_2 X_3 X_4] \cdot [X_1 + X_3 X_4 - X_1 X_3 X_4]$$

$$= X_1 X_2 + X_1 X_3 X_4 - X_1 X_2 X_3 X_4 + X_2 X_4 X_5 - X_1 X_3 X_4 X_5$$

In this expression we have introduced $\boldsymbol{X} = (X_1, X_2, X_3, X_4, X_5)$.

The correct failure probability estimate

From this, we derive the correct estimate for the probability of the two-engine system failing in a year:

$$
\begin{aligned}
&P(\text{System failure within a year}) \\
&\quad = 1 - P(\phi_2(\boldsymbol{X}) = 1) = 1 - E[\phi_2(\boldsymbol{X})] \\
&\quad = 1 - [p_1 p_2 + p_1 p_3 p_4 - p_1 p_2 p_3 p_4 + p_2 p_4 p_5 - p_1 p_3 p_4 p_5] \\
&\quad = 0.50666
\end{aligned}
$$

NOTE: With the correct approach we found that the probability of system failure for Engine 1 was 0.63373. Thus, using the correct approach Engine 2 turns out to have a significantly lower failure probability than Engine 1!!

Conclusion

If we are not careful with taking the system structure into account in the correct way, the two systems are (almost) equally reliable.

If we do the reliability analysis in the correct way, considering the structure of the system, we find from our analysis that the two-engine system is actually more reliable than the one-engine system (which was the system that was approved by Certifier).

Intuitively, this is actually quite obvious, since in the two-engine system, both engines can be used for both running the propeller and supplying power. In the one-engine system, the auxiliary engine is an irrelevant component.