6th September, 2021

STK3405/4405

Mandatory assignment 1 of 2

Submission deadline

Thursday 16nd September 2021, 14:30 in Canvas (canvas.uio.no).

Instructions

You can choose between scanning handwritten notes or typing the solution directly on a computer (for instance with LaTeX). The assignment must be submitted as a single PDF file. Scanned pages must be clearly legible. The submission must contain your name, course and assignment number.

It is expected that you give a clear presentation with all necessary explanations. Remember to include all relevant plots and figures. Students who fail the assignment, but have made a genuine effort at solving the exercises, are given a second attempt at revising their answers. All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, we may request that you give an oral account.

In exercises where you are asked to write a computer program, you need to hand in the code along with the rest of the assignment. It is important that the submitted program contains a trial run, so that it is easy to see the result of the code.

Application for postponed delivery

If you need to apply for a postponement of the submission deadline due to illness or other reasons, you have to contact the Student Administration at the Department of Mathematics (e-mail: studieinfo@math.uio.no) well before the deadline.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination.

Complete guidelines about delivery of mandatory assignments:

uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

GOOD LUCK!

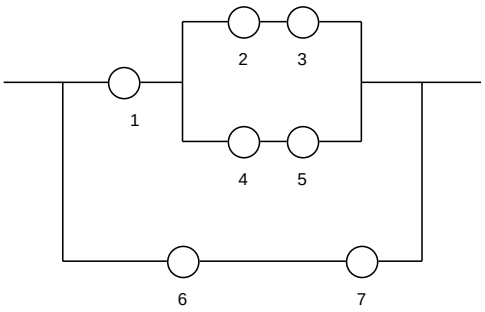

Figure 1: A binary monotone system

Problem 1. Let (C, ϕ) be the binary monotone system with component set $C = \{1, \ldots, 7\}$ and structure function ϕ shown in Figur 1.

a) Find the structure function ϕ of the system expressed as a function of the component state vector $\boldsymbol{X} = (X_1, \ldots, X_7)$.

b) Show that component 2 is relevant for the system. Is (C, ϕ) coherent? Explain your answer.

c) Let ϕ^D denote the structure function of the dual system. Find ϕ^D.

Assume that $X_1, \ldots, X_7$ are independent and that $P(X_i = 1) = p_i$, $i = 1, \ldots, 7$.

d) Find the reliability of the system expressed as a function of the vector of component reliabilities $\boldsymbol{p} = (p_1, \ldots, p_7)$.

e) Find the minimal path and cut sets of the system. [For a definition of minimal path and cut sets, see Definition 3.2.2 in the textbook.]

Problem 2. In this problem we consider a k-out-of-n-system, (C, ϕ) with component set $C = \{1, \ldots, n\}$ and structure function ϕ. Moreover, we introduce the component state vector $\boldsymbol{X} = (X_1, \ldots, X_n)$.

a) Explain why the structure function ϕ can be expressed as:

$$\phi(\boldsymbol{X}) = \mathrm{I}(\sum_{i=1}^{n} X_i \geq k).$$

where $\mathrm{I}(\cdot)$ denotes the indicator function.

b) Explain why a 1-out-of-n-system is a parallel system, while a n-out-of-n-system is a series system.

c) Show that the dual of a k-out-of-n-system is a $(n - k + 1)$-out-of-n-system.

Assume that $X_1, \ldots, X_n$ are independent and that $P(X_i = 1) = p$, $i = 1, \ldots, n$. Moreover, let $S = X_1 + \cdots + X_n$.

d) Find the distribution of S. Explain briefly why the reliability of the system, i.e., $P(\phi = 1)$ can be expressed as $P(S \geq k)$.

Assume that $k = 3$ and that $n = 6$.

e) Find the reliability of the system expressed as a function, $h = h(p)$, of the common component reliability p. [Hint: Use the result from (d).] Make a plot of $h(p)$ for $p \in [0, 1]$.