

UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in: STK3405/STK4405 — Elementary introduction to risk and reliability analysis.

Day of examination: Monday 8. December 2014.

Examination hours: 14.30–18.30.

This problem set consists of 2 pages.

Appendices: None.

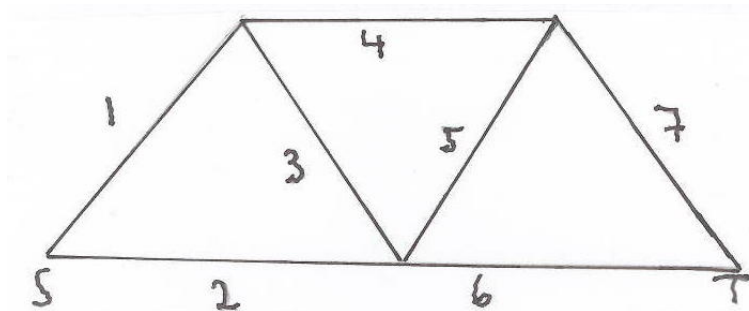
Permitted aids: Accepted calculator.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

All 11 subpoints will be equally weighted in the marking.

Problem 1

Consider the following flow network of independent component states.



- Find the minimal path and cut sets of the system.
- How many terms will we get in the best case (before we add up) by using the outmultiplication method to find the reliability of the system? Give a reason for your answer. Compare with the method based on total state enumeration.
- Find the reliability of this system as a function of the component reliabilities p_1, \dots, p_7 .
- What is reliability importance of the 4. component according to the Birnbaum measure?
- What is the corresponding structural importance of the 4. component? Show this in two ways.

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Problem 2

- a) If X_1, \dots, X_n are associated, binary, random variables, show that we then have:

$$P\left(\prod_{i=1}^n X_i = 1\right) \geq \prod_{i=1}^n P(X_i = 1)$$

$$P\left(\prod_{i=1}^n X_i = 1\right) \leq \prod_{i=1}^n P(X_i = 1)$$

- b) Based on these relations how has one raised objections against the use of reliability theory by the Norwegian Nuclear Power Commission?
- c) Let X_1, \dots, X_n be the associated component states of a monotone structure ϕ with component reliabilities p_1, \dots, p_n . Show that:

$$\prod_{i=1}^n p_i \leq P[\phi(\mathbf{X}) = 1] \leq \prod_{i=1}^n p_i$$

Association of X_1, \dots, X_n is a form of nonnegative dependence. Assume that component states are nonnegative dependent. Then give an intuitive reason for a series system to be least reliable when component states are independent, whereas a parallel system is most reliable when component states are independent.

- d) Let ϕ be a monotone system of independent component states and component reliabilities p_1, \dots, p_n and minimal path sets P_1, \dots, P_p and minimal cut sets K_1, \dots, K_k . Show in the simplest possible way that:

$$\max_{1 \leq j \leq p} \prod_{i \in P_j} p_i \leq h(\mathbf{p})$$

- e) Show that:

$$h(\mathbf{p}) \leq \min_{1 \leq j \leq k} \prod_{i \in K_j} p_i$$

by applying the lower bound in d) on the dual structure ϕ^D .

- f) Assume in addition that the structure ϕ has at least two minimal path sets. Show that then:

$$\max_{1 \leq j \leq p} \prod_{i \in P_j} p_i < h(\mathbf{p})$$

END