# Problems and Methods in Actuarial Science (STK 3505)
# Exercises 1, 9.9.2015

**Problem 1** Consider a perpetuity depending on the parameters

$m$ number of payments per year

$q$ number of payment increases per year.

More precisely, the cash flow scheme of the perpetuity is given by

| time (in years) | | | | payment |
|---|---|---|---|---|
| $0,$ | $\frac{1}{m},$ | $\cdots$ | $\frac{1}{q}-\frac{1}{m}$ | $\frac{1}{mq}$ |
| $\frac{1}{q},$ | $\frac{1}{q}+\frac{1}{m},$ | $\cdots$ | $\frac{2}{q}-\frac{1}{m}$ | $\frac{2}{mq}$ |
| $\frac{2}{q},$ | $\frac{2}{q}+\frac{1}{m},$ | $\cdots$ | $\frac{3}{q}-\frac{1}{m}$ | $\frac{3}{mq}$ |
| $\frac{3}{q},$ | $\frac{3}{q}+\frac{1}{m},$ | $\cdots$ | $\frac{4}{q}-\frac{1}{m}$ | $\frac{4}{mq}$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |

Derive a formula for the present value $\left(I^{(q)}\ddot{a}\right)^{(m)}_{\overline{\infty}|}$ of this payment stream.

**Problem 2** Let $T = n \in \mathbb{N}$ denote the duration of an annuity.
(i) Show that

$$a_{\overline{n}|} := \frac{1-v^n}{i} \text{ (PV of an immediate annuity with annual payments of 1 starting in 1),}$$

$$a^{(m)}_{\overline{n}|} := \frac{1-v^n}{i^{(m)}} \text{ (PV of an immediate annuity with payments } \frac{1}{m} \text{ } m \text{ times per year).}$$

(ii) Consider the *accumulated value* of an immediate annuity $s_{\overline{n}|}$, that is

$$s_{\overline{n}|} := (1+i)^n a_{\overline{n}|}.$$

Verify that

$$\frac{1}{a_{\overline{n}|}} = \frac{1}{s_{\overline{n}|}} + i.$$

**Problem 3 (Repayment of a debt)** Let $S$ be the present value of a debt that is to be repaid by payments $r_1, ..., r_n$ at the end of the years $k = 1, 2, ..., n$. Further denote by $S_k$ the *principal outstanding*, i.e. the value of the remaining debt at time $k$ immediately after $r_k$ has been paid (dvs. verdien av restgjeld på tid $k$ etter avdrag).

Show that

$$r_k = iS_{k-1} + (S_{k-1} - S_k), \; k = 1, ..., n,$$

that is each payment can be represented as the sum of the interest on the running debt and the *reduction of principal* (dvs. amortisajon av restgjelden).

**Problem 4** A *bond* is a contract which gives the *bondholder* (or *lender*) the right to demand from the *bond issuer* (or *borrower*) a series of future payments. The cash flow of the bond is given by a payment of $cF$ at the end of each period and a final payment $C$ at maturity $T = N$.Here $F$ denotes the *face value* and $c$ stands for the *coupon rate*. $C$ is called the *redemption value*, which usually equals $F$.The present value $PV$ of the bond can be evaluated as

$$PV = cF\frac{1-(1+i)^{-N}}{i} + C\,(1+i)^{-N}\,,$$

where $i \geq 0$ is the *yield to maturity* (effective per period) required by the lender.

Consider a company which is supposed to retire a bond issue with five annual payments of 15000. The first payment is due to December 31, 2015. In order to accumulate the funds, the company begins making annual payments of $X$ on January 1, 2006 into an account paying effective annual interest of 6%. The final payment is due to January 1, 2015. Determine $X$.

**Problem 5** John pays 98.51 for a bond that is due to mature for 100 in one year. It has coupons at 4% convertible semiannually. Calculate the annual yield rate convertible semiannually.