Problems and Methods in Actuarial Science (STK 3505)
Exercises 5, 07.10.2015

Problem 1 For a whole life insurance of 1000 issued on the life of (75), increasing net premiums, Π_k, are payable at time k, for $k = 0, 1, 2, ...$

Given:

(i) $\Pi_k = \Pi_0(1.05)^k$ for $k = 0, 1, 2, ...$

(ii) The force of mortality follows Moivre's law, that is

$$\mu_{x+t} = \frac{1}{\omega - x - t}, \ 0 < t < \omega - x$$

for $\omega = 105$.

(iii) $i = 5\%$.

Determine the net premium reserve at the end of policy year five.

Problem 2 Consider the Danish fire insurance data collected at Copenhagen Reinsurance in the period from 02/09/1987 until 02/22/1987:

Date	Loss in DKM
02/10/1987	4.128015
02/11/1987	1.076994
02/12/1987	1.502783
02/13/1987	1.021336
02/15/1987	4.500000
02/16/1987	3.107607
02/18/1987	2.254174
02/19/1987	1.757885
02/21/1987	1.398887
02/22/1987	1.252319

The above data comprise fire losses expressed in millions of Danish Krone. Assume that the claim sizes and claim arrival times are described by the Cramér-Lundberg model.

Calculate the maximum likelihood estimator of the (jump) intensity λ of the claim numbers and determine the distribution, the mean and the variance of the arrival times of the fire losses.

Problem 3 Consider a portfolio where the claims arrive according to the Cramér-Lundberg model. The *backward recurrence time* given by

$$B(t) := t - T_{N(t)}$$

is the time span between t and the last claim arrival at time $T_{N(t)}$. The process

$$F(t) := T_{N(t)+1} - t$$

is called *forward recurrence time* and measures the period of time between the time point t and the next claim arrival at time $T_{N(t)+1}$.

Show that $B(t)$ and $F(t)$ are independent.