STK 3505

Exercises 10

Problem 1 Problem 1 of Exercises 9 shows that

$$\widetilde{S}(1) \stackrel{d}{=} \sum_{i=1}^{N_\lambda} Y_i,$$

where $N_\lambda \sim Pois(\lambda)$ with $\lambda = \lambda_1 + \lambda_2 = \frac{7}{6}$. The claim sizes (Y_i) are $i.i.d$ and independent of the claim numbers N_λ. The claim size distribution is given by

$$P(Y_1 = n) = \begin{cases} 0, & n = 0 \\ 1/7 & n = 1 \\ 2/7 & n = 2, 3 \text{ or } 4 \\ 0 & n \geq 5 \end{cases}.$$

Since $P(Y_1 = 0) = 0$ and $N_\lambda \sim Pois(\lambda)$ we see from the Panjer recursion scheme (Theorem 3.3.10 in the book of T. Mikosch) that $p_n = P(\widetilde{S}(1) = n)$ is given by

$$p_n = \frac{1}{1 - aP(Y_1 = 0)} \sum_{i=1}^{n} (a + \frac{bi}{n}) P(Y_1 = i) p_{n-i}, \; n \geq 1,$$

where $p_0 = q_0 := P(N_\lambda = 0) = e^{-\lambda}$ and $a = 0, b = \lambda = 7/6$. So we have to solve the recursion

$$p_n = \sum_{i=1}^{n} \frac{7}{6} \cdot \frac{i}{n} \cdot P(Y_1 = i) \cdot p_{n-i}$$

with initial value $p_0 = P(N_\lambda = 0) = e^{-7/6}$. For $n = 0, ..., 6$ we obtain

n	0	1	2	3	4	5	6
p_n	0.3114	0.0519	0.1081	0.1213	0.1399	0.0563	0.0614

Thus

n	0	1	2	3	4	5	6
$P(\widetilde{S}(1) \leq n)$	0.3114	0.3633	0.4714	0.5927	0.7326	0.7889	0.8503

Note that

$$P(\widetilde{S}(1) \leq 18) \approx 0.999593.$$

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Prob. 2 : $S = \sum_{j=1}^{N} X_j$,

where the claim numbers $N \sim Bin(4, \frac{1}{3})$ indep. of the i.i.d fire losses (X_j)

$\longrightarrow$ $P(S \leq n)$?

for $n = 0, \ldots, 6$

$\longrightarrow$ solution : Panjer recursion scheme : $p_n := P(S = n)$

$$p_n = \frac{1}{1 - a\,P(X_1 = 0)} \sum_{i=1}^{n} \left(a + \frac{b \cdot i}{n}\right) P(X_1 = i)\, p_{n-i}, \quad n \geq 1$$

with initial value

$$p_0 = q := P(N = 0) = \binom{4}{0}\left(\frac{1}{3}\right)^0 \cdot \left(1 - \frac{1}{3}\right)^{4-0} = \left(\frac{2}{3}\right)^4$$

≈ 0.197531

$N \sim Bin(m, p)$ with $m = 4$ and $p = 3^{-1}$

Rem. 10.4.3.2 $\Longrightarrow$ or p. 123 in Mikosch $a = -p/1-p = -\frac{1}{2}$, $b = -a(m+1) = \frac{5}{2}$

$\Longrightarrow$ $p_1 \approx 0.237037$, $p_2 \approx 0.225185$, $p_3 \approx 0.128$,
$p_4 \approx 0.0997728$, $p_5 \approx 0.0547556$, $p_6 \approx 0.0335111$

$\longrightarrow$

	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$
$P(S \leq n)$	0.198	0.435	0.660	0.788	0.888	0.943	0.977

Note that

$P(S \leq 10) \approx 0.999522$