STK 3505

Exercises 7

Prob. 1: $N(t)$ renewal process as a model for the claim numbers

$\longrightarrow$ Use the Danish fire insurance data (see table) to approximate

$$Var[N(t)]$$

for $t=5$ years

Convention: Claims $X_1^{(m)}, \dots, X_n^{(m)}$ arriving on day $m \geq 0$

$\longrightarrow$ interpretation:

Claim $X_m := \sum_{i=1}^{n} X_i^{(m)}$ arrives on day m

Example: 2 fire losses at 06/06/1990

$\overset{\text{convention}}{\longrightarrow}$ $X_6 = 1.495050 + 2.042904$ on day 6 of June

Employ the asymptotic behaviour of $Var[N(t)]$, that is

$$Var[N(t)] \approx \frac{t \cdot Var[W_1]}{(E[W_1])^3} \quad \text{for } t \text{ large} \qquad (*)$$

See Prop. 2.2.10 in Mikosch

$\longrightarrow$ 1. $E[W_1]$:

$$E[W_1] \approx \frac{1}{n} \sum_{i=1}^{n} W_i =: \overline{W} \quad \text{sample mean}$$

as unbiased estimator

$$\overset{n=7}{\Longrightarrow} \quad E[W_1] \approx \frac{1}{7} \cdot 14 = 2.0 \text{ days}$$

2. $Var[W_1]$:

$$Var[W_1] \approx \frac{1}{n-1} \sum_{i=1}^{n} (W_i - \overline{W})^2$$

$$= \frac{1}{n-1} \left(\sum_{i=1}^{n} W_i^2 - n\overline{W}^2 \right) \quad \text{sample variance}$$

$$\overset{n=7}{\Longrightarrow} \quad Var[W_1] \approx \frac{1}{6} \left(2^2 + 1^2 + 2^2 + 1^2 + 4^2 + 3^2 + 1^2 - 7(2.0)^2 \right) \approx 1.3333$$

$$\overset{(*)}{\Longrightarrow} \quad Var[N(t)] \approx 5 \cdot 365 \cdot \frac{1.3333}{(2.0)^3} \approx 304.1591$$

Prob. 2: 1. $P(N(t) \geq 1050)$?

$\longrightarrow$ CLT for $N(t)$:

$N(t) \sim \mathcal{N}(\lambda \cdot t, Var[N(t)])$
approximately for large t
See Th. 9.2.5 in the manuscript

$t = 5$ years, $\lambda = \frac{1}{E[W_1]} \approx \frac{1}{2}$, $Var[N(t)] \approx 304.1591$

$$P(N(t) \geq 1050) = P\left(\frac{N(t)-\lambda t}{(Var[N(t)])^{\frac{1}{2}}} \geq \underbrace{\frac{1050-\lambda t}{(Var[N(t)])^{\frac{1}{2}}}}_{\approx 7.8841}\right)$$

$= 1 - \Phi(7.8841) \approx 0$

2. $E[S(t)]$?

$$E[S(t)] = E\left[\sum_{i=1}^{N(t)} X_i\right] = E\left[\sum_{n \geq 0}\left(\sum_{i=1}^{n} X_i\right) \cdot 1_{\{N(t)=n\}}\right]$$

$$= \sum_{n \geq 0} E\left[\left(\sum_{i=1}^{n} X_i\right) \cdot 1_{\{N(t)=n\}}\right]$$

indep

$$= \sum_{n \geq 0} \underbrace{E\left[\left(\sum_{i=1}^{n} X_i\right)\right]}_{= n \cdot E[X_1]} \cdot P(N(t)=n) = E[N(t)] \cdot E[X_1]$$

(i) $E[N(t)]$:

Elementary renewal theorem (Th. 9.2.2):

$E[N(t)] \approx \lambda \cdot t$ for large t

$\Rightarrow E[N(t)] \approx 1912.5$

(ii) $E[X_1]$:

$E[X_1] \approx \frac{1}{n} \sum_{i=1}^{n} X_i$ sample mean

$\overset{n=7}{\Longrightarrow} E[X_1] \approx 4.3074$

$\Longrightarrow E[S(t)] \approx 3930.5025$ in millions of Danish Krone